Active Contours Based On Image Laplacian Fitting Energy

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Abstract — The zero-crossings of image Laplacian edge detector is important for edge detection, which can accurately determine the edges of the image. In this paper, we propose a novel active contour model that utilizes the image Laplacian to construct an energy functional. We minimize this functional and get an item which is related to a typical image segmentation that the boundary is the zero-crossings of image Laplacian. In order to improve the ability to resist noise and extend the capture range of the force based on this energy functional, we propose another energy functional of total variation for image Laplacian. Moreover, our model is incorporated with a variational level set formulation without re-initialization proposed by Li et al. Therefore, re-initialization is unnecessary. With our model, interior contours are automatically detected with only one initial contour, which can start anywhere in the image**.** Comparisons with other major region-based models, such as the piecewise constant model (C-V model), show advantages of our model in segmentation of images with intensity in-homogeneity.

Keywords — active contours, segmentation, level sets, zero-crossings, Laplacian

Ⅰ. Introduction

Image segmentation is one of the most fundamental problems in image processing and computer vision, and various techniques have been applied for it, such as active contours in Ref.[1], [3], graph cut in Ref.[2] and so on. In this paper we focus on the Active contour models for image segmentation.

The basic idea in active contour models is to evolve a curve, subjected to an image I , to detect the objects in an image. The existing active contour models can be generally categorized into two classes: edge-based models and region-based models. The choice of them in the applications depends on different characters of the image.

In the traditional edge-based active contour models, an edge detector is used to stop the evolving curve on the boundary of the desired object. Usually, a positive, decreasing and regular edge-function $g(|\nabla I|)$ is selected, such that $\lim_{t\to\infty} g(t)=0$. For instance

$$g(|\nabla I|)=\frac{1}{1+|\nabla G_\sigma * I|^p} \qquad p\geq 1 \tag{1}$$

where $G_\sigma * I$ denotes a smooth version of I convolved with the Gaussian kernel G_σ with standard variance σ. The function $g(|\nabla I|)$ is strictly positive in homogeneous regions, and near zero on the edges.

A typical geometric edge-based active contour model is given by the following evolution equation in Ref.[4]:

$$\begin{cases} \dfrac{\partial \phi}{\partial t} = g(|\nabla I|)(\mathrm{div}(\dfrac{\nabla \phi}{|\nabla \phi|}) + v)|\nabla \phi|, \\ \qquad \text{in } [0,\infty]\times IR^2 \\ \phi(0,x) = \phi_0(x) \text{ in } IR^2 \end{cases},$$

where $g(|\nabla I|)$ is the edge-indicator function defined in Eq.(1) with $p=2$, and v is a positive constant, ϕ_0 is the initial level set function. Its zero level set curve moves in the normal direction with the velocity $g(|\nabla I|)(\mathrm{div}(\frac{\nabla \phi}{|\nabla \phi|})+v)$ and stops on the desired boundary, where g vanishes. The constant v can be interpreted as a force pushing the curve toward or outward the object boundary.

In practice, it is difficult to choose the proper balloon force term, which controls the curve to shrink or expand, such as the term $(\mathrm{div}(\frac{\nabla \phi}{|\nabla \phi|})+v)$ mentioned above. On the one hand, if the balloon force is not large enough, the evolving contour may not be able to pass some narrow parts of the objects; on the other hand, if the balloon force is too large, the evolving contour may leak through weak boundary of the object, resulting in unsatisfactory segmentation. Also, the discrete gradients are bounded and then the stopping function g never gets zero on the edges, and this may bring on the curve passing through the boundary of the object.

The region-based active contour model can solve the problems of the edge-based model mentioned above. There are many advantages over the edge-based models. First, region-based models do not utilize the gradient but the regional information; second, they are less sensitive to the location of the initial contour. One of the most popular region-based models is the Chan-Vese model in Ref.[5], which can be used to segment binary-phase image with intensity homogeneity. However, as for image with intensity in-homogeneity, the C-V model can not work well.

In this paper, we propose a novel active contour model, which can deal with the disadvantages of the models mentioned above. Based on image Laplacian, our model can accurately determine the boundary of object, and obtain a satisfactory segmentation of image with very weak edges. Besides, our model can also be applied to segment object with intensity in-homogeneity. Also, by introducing total variation for image Laplacian, our model is less sensitive to noise. Moreover, by means of incorporating a variational level set formulation without re-initialization in Ref.[6], re-initialization is unnecessary in the proposed method. Also, interior contours are automatically detected with only one initial contour, which can start anywhere in the image.

Ⅱ. Description of the Model

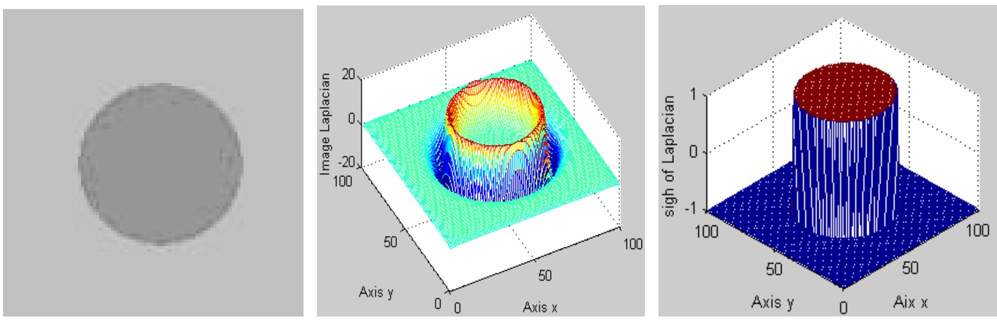
Let us first explain the idea of the model in a simple case. Assume that the image I is formed by two regions, of distinct value I^i and I^o, and the object to be detected is represented by the region with the value I^i, C_0 denotes the boundary. Now, let us consider the following energy:

$$F_1(C) + F_2(C) = \int_{outside(C)} \Delta I dx - \int_{inside(C)} \Delta I dx, x \in IR^2, \quad (2)$$

where C is a closed variable curve, and Δ is the operator of Laplacian. As we can see from Fig.1 (c), the sign of the value inside and outside of the object is opposite. In this simple case, it is obvious that C_0, which is the boundary of the object, is the solution for the minimum of the energy mentioned above:

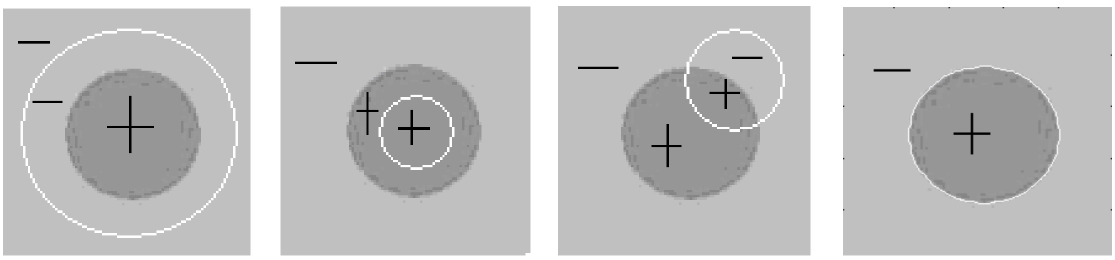
$$\inf_C \{F_1(C) + F_2(C)\} = -\int_\Omega |\Delta I| dx \approx F_1(C_0) + F_2(C_0), x \in IR^2$$

This is illustrated in Fig.2. If the curve is not on the boundary of the object, $F_1(C) > F_1(C_0), F_2(C) > F_2(C_0)$,and $F_1(C) + F_2(C) > \inf_C \{F_1(C) + F_2(C)\}$,all the possible position of the curve can been seen and the energy in Eq.(2) is minimized when $C = C_0$.



(a) Original image (b) the image Laplacian (c) the sign of the image Laplacian

Fig. 1. The original image, its Laplacian and the sign of its Laplacian



(a) $F_1(C) > F_1(C_0)$, $F_2(C) > F_2(C_0)$ (b) $F_1(C) > F_1(C_0)$, $F_2(C) > F_2(C_0)$ (c) $F_1(C) > F_1(C_0)$, $F_2(C) > F_2(C_0)$ (d) $\inf_C \{F_1(C) + F_2(C)\} = F_1(C_0) + F_2(C_0)$

Fig. 2. Consider all the possible cases in the position of contour. $+, -$ denote the sign of the value of the region of image Laplacian.

In practice, the image Laplacian is sensitive to the noise and as we can see from Fig.1(b). If the curve is far from the boundary of the object, the values of image Laplacian are near zero, that will cause the evolving force of the curve with it too small to be driven to the desired boundary. These can be partly solved by convolving the image with the Gaussian kernel G_σ with a large standard variance σ, but this will blur and distort the edges. In order to solve these problems, we utilize a more proper variant u to replace ΔI, which is obtained from minimizing the energy functional:

$$\varepsilon(u)=\int_{\Omega}|u-\Delta I|^{2}dx+\alpha\int_{\Omega\backslash C}|\nabla u|^{2}dx, x\in IR^{2} \quad (3)$$

where α is a positive constant which governs the tradeoff between the first term and the second term in the integrand. This constant should be set according to the amount of noise present in the image (more noise, increase α).

In our active contour model we will add some regularizing terms, such as the length of C. Therefore, we introduce the energy functional $\mathcal{E}^{ILF}(C,\mu,\lambda_1,\lambda_2)$ defined by:

$$\mathcal{E}^{ILF}(C,\mu,\lambda_1,\lambda_2)=\mu\bullet\text{length}(C)+\lambda_1\int_{inside(C)}u dx-\lambda_2\int_{outside(C)}u dx, x\in IR^2 \quad (4)$$

where u is obtained from minimizing the energy functional in Eq.(3), and $\mu\geq 0$, $\lambda_1\geq 0$, $\lambda_2\geq 0$ are fixed parameters.

Ⅲ. Variational Level Set Formulation of Image Laplacian Fitting model

1. Varational level set formulation of the model

In level set methods, a contour $C\subset IR^2$ is represented by the zero level set of a Lipschitz function $\phi: IR^2\rightarrow IR$, such that $\phi>0$ outside of the contour C, and $\phi<0$ inside of it. With the level set representation, the energy functional $\mathcal{E}^{ILF}(C,\mu,\lambda_1,\lambda_2)$ in Eq.(4) can be rewritten as

$$\mathcal{E}^{ILF}(\phi,\mu,\lambda_1,\lambda_2)=\mu\bullet L(\phi)+\lambda_1\int_{\Omega}uH(\phi)dx-\lambda_2\int_{\Omega}u(1-H(\phi))dx, x\in IR^2 \quad (5)$$

where H is the Heaviside function, $L(\phi)$ denotes the length of C ,which is given by

$$L(\phi)=\int_{\Omega}\delta(\phi(x))|\nabla\phi(x)|dx, x\in IR^2 \quad (6)$$

In order to ensure stable evolution of the level set function ϕ, we add the distance regularizing term in Ref. [6] to penalize the deviation of the level set function ϕ from a signed distance function, which is given by

$$p(\phi)=\int_{\Omega}\frac{1}{2}(|\nabla\phi|-1)^2 dx, x\in IR^2 \quad (7)$$

Now, we define the entire energy function:

$$F(\phi)=\mathcal{E}^{ILF}(\phi)+v\bullet p(\phi) \quad (8)$$

In practice, the Heaviside function H in Eq.(5) is approximated by a smooth function defined by

$$H_{\varepsilon}(x)=\frac{1}{2}[1+\frac{2}{\pi}\arctan(\frac{x}{\varepsilon})], x\in IR^2 \quad (9)$$

And the derivative of H_{ε} is the following smooth function

$$\delta_{\varepsilon}(x)=\frac{1}{\pi}\bullet\frac{\varepsilon}{\varepsilon^{2}+x^{2}} \tag{10}$$

By replacing H in Eq.(5) and δ in Eq.(6)with H_{ε} and δ_{ε}, the energy function $\mathcal{E}^{ILF}$ is regularized as $\mathcal{E}_{\varepsilon}^{ILF}$. We choose $\varepsilon=1.5$ for a good approximation. Therefore the energy functional in Eq.(8) can be approximated by

$$F_{\varepsilon}(\phi)=\mathcal{E}_{\varepsilon}^{ILF}(\phi)+v\bullet p(\phi) \tag{11}$$

This is the energy functional we will minimize to find the boundary of the object.

2. Gradient Decent Flow

Using the calculus of variation in Ref.[7], we minimize the energy functional in Eq.(3), and u can be found by solving the following Euler equation:

$$\alpha\Delta u-(u-\Delta I)=0 \tag{12}$$

where Δ is the Laplacian operator. This equation provides further intuition behind the formulation. We note that in a homogeneous region (where ΔI is zero), this equation is only determined by u and its Laplacian equation, and the resulting of u is interpolated among the pixels around it. Therefore, it has smoothing effect and is not sensitive to the noise.

Also, by minimizing the energy functional in Eq.(11), we could get the Euler equation, which could be written as

$$\frac{\partial F_{\varepsilon}}{\partial\phi}=-\mu\delta_{\varepsilon}(\phi)\mathrm{div}(\frac{\nabla\phi}{|\nabla\phi|})-(\lambda_1+\lambda_2)u\delta_{\varepsilon}(\phi)-v(\Delta\phi-\mathrm{div}(\frac{\nabla\phi}{|\nabla\phi|}))=0 \tag{13}$$

where δ_{ε} is the smooth Dirac function given by Eq.(10), u is the function that satisfies the Eq.(12).

Ⅳ. Implementation

1. Numerical Implementation

With steepest descent method, the following gradient flow can be obtained

$$\frac{\partial u(x,y,t)}{\partial t}=\alpha\Delta u(x,y,t)-(u(x,y,t)-\Delta I(x,y)) \tag{14}$$

To set up the iterative solution, let the indices i, j, and n correspond to x, y, and t, respectively, and the spacing between pixels be Δx and Δy and the time-step for each iteration be Δt. Then the required partial derivatives can be approximated as

$$\frac{\partial u}{\partial t}=\frac{1}{\Delta t}(u_{i,j}^{n+1}-u_{i,j}^{n})$$

$$\Delta u=\frac{1}{\Delta x\Delta y}(u_{i+1,j}+u_{i,j+1}+u_{i-1,j}+u_{i,j-1}-4u_{i,j})$$

$$\Delta I=\frac{1}{\Delta x\Delta y}(I_{i+1,j}+I_{i,j+1}+I_{i-1,j}+I_{i,j-1}-4I_{i,j})$$

Substituting these approximations into Eq.(14) gives our iterative solution to u as follows:

$$u_{i,j}^{n+1} = (1-\Delta t)u_{i,j}^{n} + \frac{\alpha \Delta t}{\Delta x \Delta y}(u_{i+1,j}^{n} + u_{i,j+1}^{n} + u_{i-1,j}^{n} + u_{i,j-1}^{n} - 4u_{i,j}^{n}) + \Delta t \Delta I \quad (15)$$

Convergence of the above iterative process is guaranteed by a standard result in the theory of numerical methods in Ref.[8]. And the Courant-Friedrichs-Lewy step-size restriction $\frac{\alpha \Delta t}{\Delta x \Delta y} \le \frac{1}{4}$ is maintained. Since Δx , Δy and α are fixed, we find that the following restriction on the time-step Δt must be maintained in order to guarantee the convergence of u :

$$\Delta t \le \frac{\Delta x \Delta y}{4\alpha}$$

The intuition behind the condition is obvious. First, convergence will be made to be faster when Δx and Δy are larger. Second, when α is large and u is expected to be a smoother item, the convergence rate will be slower.

Also, Eq.(13) can be solved by this method, and we finally obtain the evolution equation as

$$\frac{\partial \phi}{\partial t} = \mu \delta_{\varepsilon}(\phi)\mathrm{div}(\frac{\nabla \phi}{|\nabla \phi|}) + (\lambda_1 + \lambda_2)u\delta_{\varepsilon}(\phi) + v(\Delta \phi - \mathrm{div}(\frac{\nabla \phi}{|\nabla \phi|}))$$

Let $G(\phi) = \mu \delta_{\varepsilon}(\phi)\mathrm{div}(\frac{\nabla \phi}{|\nabla \phi|}) + (\lambda_1 + \lambda_2)u\delta_{\varepsilon}(\phi) + v(\Delta \phi - \mathrm{div}(\frac{\nabla \phi}{|\nabla \phi|}))$ and $\lambda = \lambda_1 + \lambda_2$, then the equation above can be written as:

$$\frac{\partial \phi}{\partial t} = G(\phi) \quad (16)$$

Because of the diffusion item introduced by our penalizing energy functional, we no longer need the upwind scheme in Ref.[8] as in the traditional level set methods. Instead, the entire spatial derivative $\frac{\partial \phi}{\partial x}$ and $\frac{\partial \phi}{\partial y}$ are approximated by the central difference. The approximation of Eq.(16) can be written as

$$\frac{\phi_{i,j}^{n+1} - \phi_{i,j}^{n}}{\tau} = G(\phi_{i,j}^{n}) \quad (17)$$

where τ is the time step. Then we get the approximation iterative equation as

$$\phi_{i,j}^{n+1} = \phi_{i,j}^{n} + \tau G(\phi_{i,j}^{n}) \quad (18)$$

In order to maintain stable level set evolution, the time-step should satisfy $v\tau < \frac{1}{4}$, as pointed out by

Ref.[6].

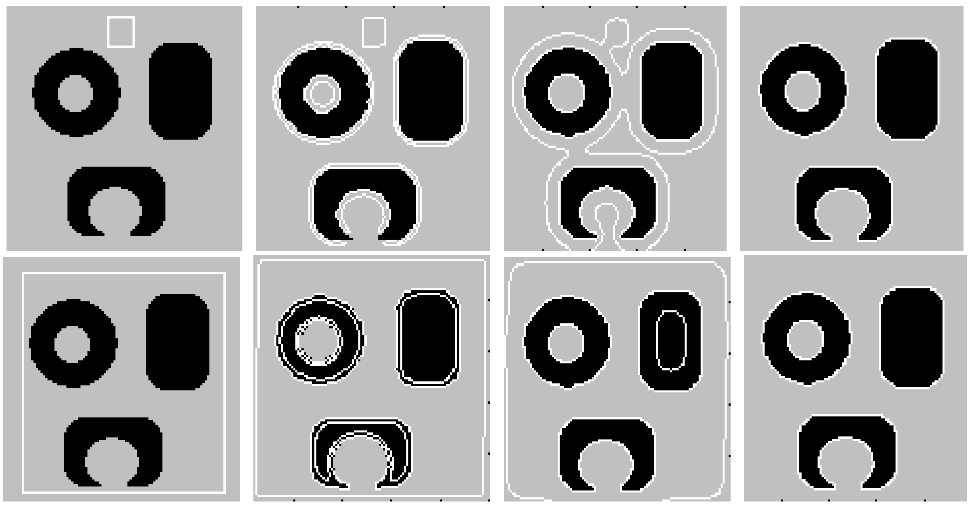
2. Initialization of Level Set function

In traditional level set methods, it is necessary to initialize the level set function as a signed distance function, but in our model, as introducing the penalizing energy functional of signed distance function in Eq.(7), we could flexibly initialize the level set function ϕ. In particular, we can simply initialize ϕ as a binary function, which takes a positive constant ρ in a region Ω_0, and $-\rho$ outside of Ω_0, where Ω_0 can be any arbitrarily given subset in the image domain Ω.

Ⅴ. Experimental Results

We validate our model with synthetic images and real images of different modalities. We select the same parameters of $\rho=1, v=0.4, \mu=5, \lambda=3.5,$ and $\alpha=2.5,$ time-step $\Delta t=0.08$ and $\tau=0.5$, $\Delta x=1$, $\Delta y=1$ for most of the images in this paper. If the image is too noisy, we select a large α. In order to get a smoother image, we first utilize a Gaussian kernel G_σ to convolve with the image, where σ is the standard variance. We use the parameter of σ=2.

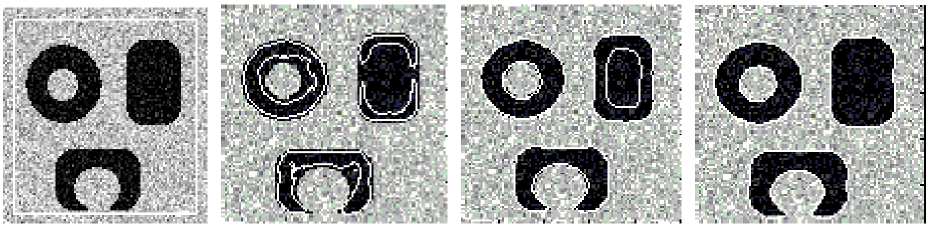
In Fig. 3 we show how our model works on a synthetic image with different initial contours, objects with different shapes, convexities and an interior contour. The results reveal that our model is not sensitive to the initialization of the contour; different initial contour will get the same result. Also, our model can automatically detect interior contours of object, without necessity of initial contour around objects or more initial contours.



(a) Initial contour (b) 1 iteration (c) 6 iterations (d) final results

Fig. 3. Application to different objects with various convexities, different shapes and an interior contour, and with different initial contours.

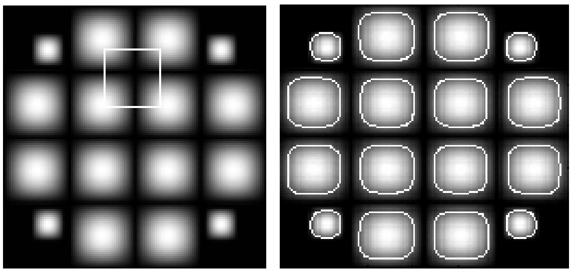
Fig. 4. shows how our model works in an image with noise. The result reveals that the edges of the objects are maintained, because we utilize u in Eq.(14) instead of image Laplacian ΔI, which is able to smooth the image Laplacian without blurring the edge of image, In this experiment, we choose the parameter of $\alpha=4.5$ .It is worth pointing out that the new contours can emerge during the process of evolution only after one iteration, as depicted in Fig.4.(b).



(a) Initial contour. (b) 1 iteration (c) 4 iterations (d) 10 iterations

Fig.4. Application to a noisy synthetic image

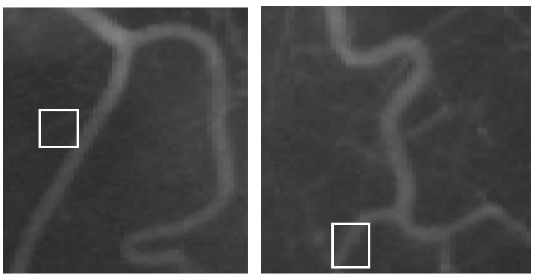
Fig. 5. shows the result of our model for a synthetic image with blurred object. As we can see from Fig.5(b), the contours accurately stop on the boundary of the objects.



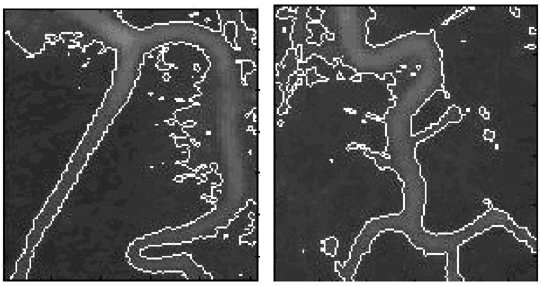
(a) Initial contour (b) The result of convolution

Fig. 5. Application to a synthetic image with weak edges.

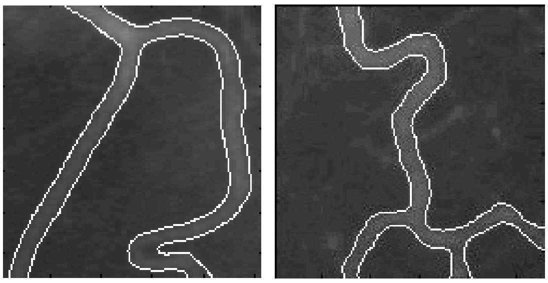
Fig. 6. illustrates the comparisons between C-V model and our model to segment two real vessel images. As we can see from Fig. 6, the intensities of the two images are inhomogeneous. As the C-V model utilizes the statistic homogeneity of the image to control the evolution of the curve, it can't work well. Our model have advantages over it, as our model utilizes the image Laplacian to control the evolution, although the intensity of the image is of statistic in-homogeneity, the image Laplacian inside of the object interested and outside of it have inverse signs,. As we utilize u in Eq.(14) instead of image Laplacian ΔI, we can get smoother and more accurate contours.



(a) Initial contours



(b) The results of C-V model



(c) The results of our model

Fig. 6. The comparisons between C-V model and our model with application to two real vessel images.

Ⅵ.Conclusion

In this paper, we propose a novel active contour model for image segmentation, which is based on image Laplacian. The proposed model utilizes the zero-crossings of image Laplacian to segment object of interest, and can accurately determine the boundary of the object. It is able to segment image with intensity in-homogeneity and weak edge objects with promising results. Also, we propose a novel item, which gets from minimizing the energy functional of Eq.(3)is obtained through a total variation formulation, to replace image Laplacian, and it is less sensitive to the noise. Furthermore, no re-initialization is necessary in our model and we could get a simple and fast implementation. Comparison with the C-V model shows that our model has an advantage of segmenting images with intensity in-homogeneity. Finally, the interior contours can be automatically detected with only one starting contour and the initial contour can be anywhere in the image.

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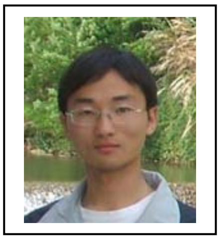

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