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Stable adaptive fuzzy control for MIMO nonlinear systems

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ABSTRACT

In this paper, an indirect adaptive fuzzy control scheme is presented for a class of multi-input and multi-output (MIMO) nonlinear systems whose dynamics are poorly understood. Within this scheme, fuzzy systems are employed to approximate the plant's unknown dynamics. In order to overcome the controller singularity problem, the estimated gain matrix is decomposed into the product of one diagonal matrix and two orthogonal matrices, a robustifying control term is used to compensate for the lumped errors, and all parameter adaptive laws and robustifying control term are derived based on Lyapunov stability analysis. The proposed scheme guarantees that all the signals in the resulting closed-loop system are uniformly ultimately bounded (UUB). Moreover, the tracking errors can be made small enough if the designed parameter is chosen to be sufficiently large. A simulation example is used to demonstrate the effectiveness of the proposed control scheme.

1. Introduction

In practical control engineering, fuzzy system-based adaptive control methodologies have received much attention, emerging as promising approaches for controlling highly uncertain and nonlinear dynamical systems. Based on the universal approximation theorem [1], during the last two decades, several adaptive fuzzy control schemes have been developed for a class of single-input single-output (SISO) nonlinear uncertain systems [2–6], and multi-input multi-output (MIMO) nonlinear uncertain systems are investigated in [7–13]. Stability analysis in such schemes is performed by using the Lyapunov synthesis method.

In order to meet control objectives, conceptually, there exist two distinct approaches to design a fuzzy adaptive control system: direct and indirect schemes. In the direct method, a fuzzy system is used to describe the control action and the parameters of the fuzzy system are adjusted directly to meet the control objective [1,4,5,12,13]. Unlike the direct schemes, the indirect adaptive approach uses fuzzy systems to estimate the plant dynamics and then a control law is designed based on these estimates [1–3,6–11,13]. In indirect adaptive schemes, the possible controller singularity problem usually meet. To avoid this problem, for MIMO systems, in [7,8], the authors suggest using a projection algorithm to keep the estimated parameters inside a feasible set, but this solution has some disadvantages [10,11]. In [9,13] the authors do not take account of the controller singularity problem, they implicitly assume that the estimated control gain matrix is always nonsingular. In [11], for the certainty control term, authors use the regularized inverse of the estimated control gain matrix instead of its inverse to avoid the possible singularity problem, and design a robustifying term to compensate for the approximation errors, but it is possible for the robustifying term not to be well-defined. Another way to avoid this problem is to use the direct adaptive control schemes [12]. However, this approach seems to require that the gain matrix satisfies more restrictive assumptions [11].

Moreover, a key assumption in the developed adaptive fuzzy control schemes [11,12] is that the control gain matrix is positive definite, however, in [11,12], the main care the authors take into account is the sign of the gain matrix instead

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of the symmetry. In this paper, motivated by the symmetric matrix decomposition technique [14], an indirect adaptive fuzzy control scheme is developed for a class of uncertain MIMO nonlinear systems. Within this scheme, the fuzzy systems are used to approximate the plant's unknown dynamics, in order to avoid the possible controller singularity problem, the estimated symmetric gain matrix is decomposed into the product of one diagonal matrix and two orthogonal matrices, and a robust controller is used to compensate the lumped errors. The proposed design scheme guarantees that all the signals in the resulting closed-loop system are UUB. Moreover, the tracking errors can be reduced by adjusting the value of the designed parameter.

The rest of the paper is organized as follows. In Section 2, we describe the plant dynamics, control objectives, and a brief description of fuzzy systems. In Section 3 the suggested indirect adaptive fuzzy control schemes are presented while in Section 4, simulation results are provided to demonstrate the effectiveness of the method. Finally, conclusions are drawn in Section 5.

Throughout this paper, $\|.\|$ indicates the Euclidean norm.

2. Problem formulation and preliminaries

Consider a MIMO nonlinear dynamic system represented by the following form

$$\begin{aligned} y_1^{(r_1)} &= f_1(x) + \sum_{j=1}^{p} g_{1j}(x)u_j, \\ &\vdots \\ y_i^{(r_i)} &= f_i(x) + \sum_{j=1}^{p} g_{ij}(x)u_j \end{aligned} \tag{1}$$

where $x = [y_1, \dot{y}_1, \ldots, y_1^{(r_1-1)}, \ldots, y_p, \dot{y}_p, \ldots, y_p^{(r_p-1)}]^T \in R^l$ is the system state vector, which is assumed available for measurement and $l = \sum_{i=1}^{p} r_i$, $u = [u_1, \ldots, u_p]^T \in R^p$ and $y = [y_1, \ldots, y_p]^T \in R^p$ are the system input vector and output vector, respectively, and $f_i(x)$, $i = 1, 2, \ldots, p$ and $g_{ij}(x)$, $i, j = 1, 2, \ldots, p$, are continuous unknown smooth nonlinear functions.

Let us denote

$$y^{(r)} = [y_1^{(r_1)}, \ldots, y_p^{(r_p)}]^T,$$
$$F(x) = [f_1(x), \ldots, f_p(x)]^T,$$
$$G(x) = \begin{bmatrix} g_{11}(x) & \cdots & g_{1p}(x) \\ \vdots & \ddots & \vdots \\ g_{p1}(x) & \cdots & g_{pp}(x) \end{bmatrix}.$$

Then, Eq. (1) can be written in the following compact form

$$y^{(r)} = F(x) + G(x)u \tag{2}$$

where $F(x) \in R^p$ and $G(x) \in R^{p\times p}$.

The controllability of (1) requires that $G(x)$ is nonsingular for all $x \in U \subset R^l$, accordingly, throughout this paper we make the following assumption:

Assumption 1 (*[11–13]*). For all $x \in U, G(x)$ is positive definite, then there exists unknown $\sigma_0 > 0, \sigma_0 \in R$ such that $G(x) \geq \sigma_0 I_p$, here I_p is the $p \times p$ identity matrix.

The objective of this paper is to design an indirect adaptive fuzzy controller $u(t)$ such that the system output y follows the reference signal $y_m = [y_{m1}, y_{m2}, \ldots, y_{mp}]^T$, i.e., the tracking error $e_i(t) = y_{mi}(t) - y_i(t) = 0$ $(i = 1, \ldots, p)$, while all the signals in the derived closed-loop system remain bounded.

Let us define the filtered tracking errors as

$$\begin{aligned} s_i &= \left(\frac{\mathrm{d}}{\mathrm{d}t} + \lambda_1\right)^{r_1-1} e_1(t), \quad \lambda_1 > 0 \\ &\vdots \\ s_p &= \left(\frac{\mathrm{d}}{\mathrm{d}t} + \lambda_p\right)^{r_p-1} e_p(t), \quad \lambda_p > 0. \end{aligned} \tag{3}$$

Eq. (3) can be written as follows

$$s_i = e_i^{(r_1-1)} + (r_i - 1)\lambda_i e_i^{(r_i-2)} + \cdots + (r_i - 1)\lambda_i^{r_i-2}\dot{e}_i + \lambda_i^{r_i-1} e_i \quad (i = 1, 2, \ldots, p). \tag{4}$$

Remark 1. From (4), if we choose the appropriate λ_i such that the roots of the equation $H_i(s) = s^{r_1-1} + (r_i - 1)\lambda_i s^{r_i-2} + \cdots + (r_i - 1)\lambda_i^{r_i-2} s + \lambda_i^{r_i-1} = 0$ $(i = 1, 2, \ldots, p)$ are all in the left-half complex plane, it follows that $e_i(t) \to 0$ asymptotically as $s_i(t) \to 0$. Thus, the problem of tracking the r_i-dimensional vector y_{mi} can be replaced by a 1st-*order* stabilization problem in the scalar s_i. Moreover, if $|s_i(t)| \le \Psi_i$, $\forall t \ge 0$, then $|e_i(t)| \le \frac{\Psi_i}{\lambda_i^{r_1-1}}$, $\forall t \ge 0$, within a short time-constant $(r_i - 1)/\lambda_i$ $(i = 1, 2, \ldots, p)$ [15].

So, the objective of this paper becomes the design of a control law to force the filtered tracking error $s_i(t) \to 0$, or to be ultimately bounded.

From (3), it is obvious that the expression of s_i contains $e_i^{r_i-1}$, one only needs to differentiate s_i once for the input u_j to appear. Differentiating s_i with respect to time yields

$$\begin{aligned} \dot{s}_1 &= v_1 - f_1(x) - \sum_{j=1}^{p} g_{1j}(x) u_j \\ &\vdots \\ \dot{s}_p &= v_p - f_p(x) - \sum_{j=1}^{p} g_{pj}(x) u_j \end{aligned} \tag{5}$$

where

$$\begin{aligned} v_1 &= y_{m1}^{(r_1)} + \beta_{1,r_1-1} e_1^{(r_1-1)} + \cdots + \beta_{1,1}\dot{e}_1 \\ &\vdots \\ v_p &= y_{mp}^{(r_p)} + \beta_{p,r_p-1} e_p^{(r_p-1)} + \cdots + \beta_{p,1}\dot{e}_p \end{aligned} \tag{6}$$

with

$$\beta_{i,j} = \frac{(r_i - 1)!}{(r_i - j)!(j - 1)!}\lambda_i^{r_i-j}, \quad i = 1, \ldots, p, \; j = 1, \ldots, r_i - 1.$$

Denote

$$s = [s_1, \ldots, s_p]^T$$
$$v = [v_1, \ldots, v_p]^T.$$

Then Eq. (5) can be written in the following form

$$\dot{s} = v - F(x) - G(x)u. \tag{7}$$

If the nonlinear functions $f_i(x)$ and $g_{ij}(x)$ are known, then the following control law

$$u = G^{-1}(x)(-F(x) + v + K_0 s) \tag{8}$$

where $K_0 = \mathrm{diag}[k_{01}, \ldots, k_{0p}]$ with $k_{0i} > 0$, can be used to meet the control objective. Indeed, substituting (8) into (7), we get

$$\dot{s} = -K_0 s. \tag{9}$$

From which we can conclude that $s_i \to 0$ as $t \to \infty$.

From the aforementioned analysis, we know that the control law (8) is easily implemented in the case where $F(x)$ and $G(x)$ are known. However, in this paper, these nonlinear functions are unknown, the above control law (8) cannot be implemented. In this case, we assume that they can be approximated by the fuzzy systems. In the following, the fuzzy systems considered in this paper are discussed briefly. The used fuzzy systems are characterized by a set of fuzzy IF-THEN rules in the following form [1]

$R^{(l)}$: IF x_1 is F_1^l and $\cdots$ and $x_n$is F_n^l, THEN y is G^l

where $x = [x_1, \ldots, x_n]^T$ and y are the input and output of the fuzzy logic system, respectively, F_i^l and G^l are fuzzy sets, for $l = 1, \ldots, m$.

By using the strategy of singleton fuzzification, product inference and center-average defuzzification, the output of the fuzzy system is given as follows

$$y=\frac{\sum_{j=1}^{m} y^j\left(\prod_{i=1}^{n}\mu_{F_i^j}(x_i)\right)}{\sum_{j=1}^{m}\prod_{i=1}^{n}\mu_{F_i^j}(x_i)} \tag{10}$$

where y^j is the point at which the membership function of G^l achieves its maximum value. By introducing the concept of fuzzy basis function vector $\xi(x)$, Eq. (10) can be rewritten as

$$y(x)=\hat{f}(x|\theta)=\theta^T\xi(x) \tag{11}$$

where $\theta=[y^1,\dots,y^m]^T$, $\xi(x)=[\xi^1(x),\dots,\xi^m(x)]^T$ with $\xi^j(x)=\frac{\prod_{i=1}^{n}\mu_{F_i^j}(x_i)}{\sum_{j=1}^{m}\prod_{i=1}^{n}\mu_{F_i^j}(x_i)}$.

According to the universal approximation theorem [1], the fuzzy system (11) is able to approximate any continuous nonlinear function on a compact set to an arbitrary degree of accuracy provided that enough number of rules are considered. In following, it is assumed that the structure of the fuzzy system and the fuzzy basis function parameters are properly specified in advance by the designer. This means that the designer decision is needed to determine the structure of the fuzzy system, and the consequent parameters must be calculated by adaptive laws.

3. Design of indirect adaptive fuzzy control and stability analysis

Since $f_i(x)$ and $g_{ij}(x)$ in (1) are unknown, we assume that they can be approximated by the fuzzy systems in the form of (11) as follows

$$\hat{f}_i(x|\theta_{fi})=\theta_{fi}^T\xi_{fi}(x),\quad i=1,\dots,p,$$

$$\hat{g}_{ij}(x|\theta_{gij})=\theta_{gij}^T\xi_{gij}(x),\quad i,j=1,\dots,p$$

where $\xi_{fi}(x)$ and $\xi_{gij}(x)$ are fuzzy basis function vectors, θ_{fi} and θ_{gij} are parameter vectors of each fuzzy system designed later.

Denote

$$\hat{F}(x|\theta_f)=[\hat{f}_1(x|\theta_{f1}),\dots,\hat{f}_p(x|\theta_{fp})]^T$$

and from Assumption 1, we know that $G(x)$ is symmetric, thus it is reasonable to assume that its fuzzy approximation $\hat{G}(x|\theta_g)$ is symmetric, too, the matrix $\hat{G}(x|\theta_g)$ can be denoted as

$$\hat{G}(x|\theta_g)=\begin{bmatrix}\hat{g}_{11}(x|\theta_{g11}) & \hat{g}_{12}(x|\theta_{g12}) & \cdots & \hat{g}_{1p}(x|\theta_{g1p})\\ \hat{g}_{12}(x|\theta_{g12}) & \hat{g}_{22}(x|\theta_{g22}) & \cdots & \hat{g}_{2p}(x|\theta_{g2p})\\ \vdots & \ddots & \vdots & \vdots\\ \hat{g}_{1p}(x|\theta_{g1p}) & \hat{g}_{2p}(x|\theta_{g2p}) & \cdots & \hat{g}_{pp}(x|\theta_{gpp})\end{bmatrix}$$

Remark 2. Compared with this paper, in [11–13], the gain matrix $G(x)$ is positive definite symmetric, however, the symmetry of the estimated gain matrix $\hat{G}(x|\theta_g)$ is not considered.

Now, let us consider a certainty control law as follows

$$u_c=\hat{G}^{-1}(x|\theta_g)(-\hat{F}(x|\theta_f)+v+K_0s). \tag{12}$$

This control law results from (8) by using the adaptive fuzzy approximations $\hat{F}(x|\theta_f)$ and $\hat{G}(x|\theta_g)$ instead of the functions $F(x)$ and $G(x)$, respectively.

Since the matrix $\hat{G}(x|\theta_g)$ is generated online by the estimation of the parameters θ_g, the control law (12) is not well-defined when the estimated gain matrix $\hat{G}(x|\theta_g)$ is singular. In order to overcome this problem, we use the symmetric matrix decomposition technique. Since matrix $\hat{G}(x|\theta_g)$ is symmetric, it can be decomposed as follows [14]

$$\hat{G}(x|\theta_g)=P^{-1}D_\lambda P \tag{13}$$

where P is a orthogonal matrix, $D_\lambda=\mathrm{diag}[\lambda_1,\dots,\lambda_p]$, here λ_i is the characteristic root of matrix $\hat{G}(x|\theta_g)$.

We modify the equivalent control law (12) as follows

$$u_c=[\hat{G}(x|\theta_g)+P^{-1}D_{\lambda\varepsilon}P]^{-1}(-\hat{F}(x|\theta_f)+v+K_0s) \tag{14}$$

where $D_{\lambda\varepsilon}=\mathrm{diag}[\varepsilon_1\mathrm{sign}(\lambda_1),\dots,\varepsilon_p\mathrm{sign}(\lambda_p)]$ with $\varepsilon_i>0$, and $\mathrm{sign}(\lambda_i)=\begin{cases}1 & \text{if } \lambda_i\geq 0\\ -1 & \text{if } \lambda_i<0\end{cases}$.

Since $\hat{G}(x|\theta_g) + P^{-1}D_{\lambda\varepsilon}P = P^{-1}(D_\lambda + D_{\lambda\varepsilon})P$, it is obvious that the matrix $\hat{G}(x|\theta_g) + P^{-1}D_{\lambda\varepsilon}P$ is nonsingular, therefore the controller (14) is always well-defined.

Due to $F(x)$ being approximated by $\hat{F}(x|\theta_f)$, $G(x)$ is approximated by $\hat{G}(x|\theta_g) + P^{-1}D_{\lambda\varepsilon}P$, so the approximation errors are unavoidable. In order to compensate for these errors, we append to the controller (14) a robust control term u_r

$$u = u_c + u_r \tag{15}$$

where u_r is to be designed later.

Substituting (15) into (7) yields

$$\dot{s} = -K_0 s + (\hat{F}(x|\theta_{fi}) - F(x)) + (\hat{G}(x|\theta_{gij}) - G(x))u_c + P^{-1}D_{\lambda\varepsilon}Pu - Gu_r. \tag{16}$$

Let us define the optimal approximation parameters θ_{fi}^* and θ_{gij}^* as follows:

$$\theta_{fi}^* = \arg\min_{\theta_{fi}\in\Omega_{fi}}[\sup_{x\in U}|\hat{f}_i(x|\theta_{fi}) - f_i(x)|]$$

$$\theta_{gij}^* = \arg\min_{\theta_{gij}\in\Omega_{gij}}[\sup_{x\in U}|\hat{g}_{ij}(x|\theta_{gij}) - g_{ij}(x)|]$$

where Ω_{fi} and Ω_{gij} are the compact set of allowable controller parameters. Define the parameter errors

$$\phi_{fi} = \theta_{fi} - \theta_{fi}^*, \phi_{gij} = \theta_{gij} - \theta_{gij}^*,$$

and

$$\omega_{fi} = \hat{f}_i(x|\theta_{fi}^*) - f_i(x),$$

$$\omega_{gij} = \hat{g}_{ij}(x|\theta_{gij}^*) - g_{ij}(x)$$

as the minimum approximation errors.

In this paper, we assume that the used fuzzy system does not violate the universal approximation theorem [1] on the compact set U, which is assumed large enough so that state variables remain within U under closed-loop control. So it is reasonable to assume that the minimum approximation errors are bounded for all $x \in U$, accordingly, we can make the following assumption:

Assumption 2. For $i, j = 1, 2, \ldots, p$, ω_{fi}, ω_{gij} are bounded, respectively.

With above definition, Eq. (16) can be written as follows

$$\dot{s} = -K_0 s + \Xi_f \Phi_f + \Xi_g \Phi_g + P^{-1}D_{\lambda\varepsilon}Pu_c + \Omega_{ufg}\omega_{fg} - Gu_r \tag{17}$$

where

$$\Xi_f = \begin{bmatrix} \xi_{f1}^T(x) & & & \\ & \xi_{f2}^T(x) & & \\ & & \ddots & \\ & & & \xi_{fp}^T(x) \end{bmatrix},$$

$$\Xi_g = \begin{bmatrix} \Xi_{g11} & & & \\ & \Xi_{g22} & & \\ & & \ddots & \\ & & & \Xi_{gpp} \end{bmatrix}$$

with

$$\Xi_{g11} = [\xi_{g11}^T(x)u_{c1}, \ldots, \xi_{g1p}^T(x)u_{cp}],$$

$$\Xi_{g22} = [\xi_{g12}^T(x)u_{c1}, \ldots, \xi_{g2p}^T(x)u_{cp}],$$

$$\Xi_{gpp} = [\xi_{g1p}^T(x)u_{c1}, \ldots, \xi_{gpp}^T(x)u_{cp}]$$

$$\Phi_f = [\phi_{f1}^T, \phi_{f2}^T, \ldots, \phi_{fp}^T]^T,$$

$$\Phi_g = [\phi_{g11}^T, \ldots, \phi_{g1p}^T, \phi_{g12}^T, \ldots, \phi_{g2p}^T, \phi_{g1p}^T, \ldots, \phi_{gpp}^T]^T$$

$$\omega_{fg} = [\omega_{f1}, \omega_{g11}, \ldots, \omega_{g1p}, \ldots, \omega_{fp}, \omega_{g1p}, \ldots, \omega_{gpp}]^T$$

$$\Omega_{ufg} = \begin{bmatrix} u_{fg} & & & \\ & u_{fg} & & \\ & & \ddots & \\ & & & u_{fg} \end{bmatrix}$$

with $u_{fg} = [1, u_{c1}, \ldots, u_{cp}]$.

In order to compensate for the lumped error terms in (17), the robust term u_r is designed as

$$u_r = \beta s[\|D_{\lambda\varepsilon}\|^2 \|u_c\|^2 + \|\Omega_{ufg}\|^2] \tag{18}$$

where β is a positive designed parameter.

Denote

$$\Theta_f = [\theta_{f1}^T, \dots, \theta_{fp}^T]^T,$$
$$\Theta_g = [\theta_{g11}^T, \dots, \theta_{g1p}^T, \theta_{g12}^T, \dots, \theta_{g2p}^T, \theta_{g1p}^T, \dots, \theta_{gpp}^T]^T.$$

The adaptation laws for Θ_f and Θ_g are defined as follows:

$$\dot{\Theta}_f = -\gamma_f \Xi_f^T s, \tag{19}$$

$$\dot{\Theta}_g = -\gamma_g \Xi_g^T s \tag{20}$$

where $\gamma_f > 0$, $\gamma_g > 0$.

Remark 3. From (20), if $i < j$, then there exist two values for θ_{gij}, them are generally unequal, so we average these two values to obtain the true value of θ_{gij}.

The property of the designed control scheme is summarized by the following theorem.

Theorem 1. *Given the plant defined by (1) satisfying Assumptions 1 and 2, the control law (15) with adaptation law (19)–(20) will ensure that all signals in the closed-loop system are uniformly ultimately bounded, and the tracking errors can be made small enough if the designed parameter β is chosen to be sufficiently large.*

Proof. Consider the following Lyapunov function candidate:

$$V = \frac{1}{2} s^T s + \frac{1}{2\gamma_f} \Phi_f^T \Phi_f + \frac{1}{2\gamma_g} \Phi_g^T \Phi_g. \tag{21}$$

The time derivative of V is

$$\begin{aligned}\dot{V} &= s^T \dot{s} + \frac{1}{\gamma_f} \dot{\Phi}_f^T \Phi_f + \frac{1}{\gamma_g} \dot{\Phi}_g^T \Phi_g \\ &= s^T(-K_0 s + \Xi_f \Phi_f + \Xi_g \Phi_g + P^{-1} D_{\lambda\varepsilon} P u_c + \Omega_{ufg}\omega_{fg} - G u_r) + \frac{1}{\gamma_f} \dot{\Phi}_f^T \Phi_f + \frac{1}{\gamma_g} \dot{\Phi}_g^T \Phi_g. \end{aligned} \tag{22}$$

Using (19) and (20), we have

$$s^T \Xi_f \Phi_f + \frac{1}{\gamma_f} \dot{\Phi}_f^T \Phi_f + s^T \Xi_g \Phi_g + \frac{1}{\gamma_g} \dot{\Phi}_g^T \Phi_g = 0. \tag{23}$$

Since matrix P is an orthogonal matrix, $\|P^{-1} D_{\lambda\varepsilon} P\| = \|D_{\lambda\varepsilon}\|$. With (18), we have

$$s^T(P^{-1} D_{\lambda\varepsilon} P u_c + \Omega_{ufg}\omega_{fg} - G u_r) \le \|s\|[\|D_{\lambda\varepsilon}\|\,\|u_c\| + \|s\|\,\|\Omega_{ufg}\|\,\|\omega_{fg}\|] - \beta\sigma_0 \|s\|^2 [\|D_{\lambda\varepsilon}\|^2 \|u_c\|^2 + \|\Omega_{ufg}\|^2]. \tag{24}$$

Here, the inequality $s^T G s \ge \sigma_o \|s\|^2$ is used which is true because $G(x)$ satisfies Assumption 1.

Using the inequality $2ab - b^2 \le a^2$, we have

$$\|s\|\,\|D_{\lambda\varepsilon}\|\,\|u_c\| - \beta\sigma_0 \|s\|^2 \|D_{\lambda\varepsilon}\|^2 \|u_c\|^2 \le \frac{1}{4\beta\sigma_0} \tag{25}$$

$$\|s\|\,\|\Omega_{ufg}\|\,\|\omega_{fg}\| - \beta\sigma_0 \|s\|^2 \|\Omega_{ufg}\|^2 \le \frac{\|\omega_{fg}\|^2}{4\beta\sigma_0} \tag{26}$$

using (25) and (26), (24) becomes

$$s^T(P^{-1} D_{\lambda\varepsilon} P u_c + \Omega_{ufg}\omega_{fg} - G u_r) \le \frac{1}{4\beta\sigma_0}[1 + \|\omega_{fg}\|^2]. \tag{27}$$

Substituting (23) and (27) into (22) yields

$$\dot{V} \le -\min_{1\le i\le p}\{K_{0i}\}\|s\|^2 + \frac{1}{4\beta\sigma_0}[1 + \|\omega_{fg}\|^2]. \tag{28}$$

From Assumption 2, we know that $\|\omega_{fg}\|$ is bounded, so the constant $\rho_{fg} > 0$ exists such that $\|\omega_{fg}\|^2 \le \rho_{fg}$. Then, one can guarantee that $\dot{V}$ is negative as long as s is outside the compact set Ω defined as

$$\Omega = \{s|\ \|s\| \le \Psi\} \tag{29}$$

where $\Psi = \sqrt{\frac{1+\rho_{fg}}{4\beta\sigma_0 \min_{1\le i\le p}\{K_{0i}\}}}$.

According to a standard Lyapunov theorem, we conclude that s, Φ_f, Φ_g are all uniformly ultimately bounded, and s will converge to Ω. Therefore, s_i will converge to Ω. Moreover, we can obtain that [15]

$$|e_i(t)| \le \frac{\Psi}{\lambda_i^{r_1-1}}, \quad \forall t \ge 0 \tag{30}$$

within a short time-constant $(r_i - 1)/\lambda_i$ $(i = 1, 2, \ldots, p)$.

From (30), it is obvious that tracking error $e_i(t)$ can be made small enough by properly choosing sufficiently large designed parameter β. □

4. Simulation

Consider a two-link rigid robot manipulator moving in a horizontal plane. The dynamic equations of such a system are given by [11,12,15]

$$\begin{bmatrix}\ddot{q}_1\\ \ddot{q}_2\end{bmatrix} = \begin{bmatrix}M_{11} & M_{12}\\ M_{21} & M_{22}\end{bmatrix}^{-1} \left\{\begin{bmatrix}u_1\\ u_2\end{bmatrix} - \begin{bmatrix}-h\dot{q}_2 & -h(\dot{q}_1+\dot{q}_2)\\ h\dot{q}_1 & 0\end{bmatrix}\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\end{bmatrix}\right\}$$

where

$$\begin{aligned}
M_{11} &= a_1 + 2a_3\cos(q_2) + 2a_4\sin(q_2),\\
M_{22} &= a_2,\\
M_{12} &= M_{21} = a_2 + a_3\cos(q_2) + a_4\sin(q_2),\\
h &= a_3\sin(q_2) - a_4\cos(q_2)
\end{aligned}$$

with

$$\begin{aligned}
a_1 &= I_1 + m_1 l_{c1}^2 + I_e + m_e l_{ce}^2 + m_e l_1^2,\\
a_2 &= I_e + m_e l_{ce}^2,\\
a_3 &= m_e l_1 l_{ce}\cos(\delta_e),\\
a_4 &= m_e l_1 l_{ce}\sin(\delta_e).
\end{aligned}$$

In the simulation, the following parameter values are used: $m_1 = 1$, $m_e = 2$, $l_1 = 1$, $l_{c1} = 0.5$, $l_{ce} = 0.6$, $I_1 = 0.12$, $I_e = 0.25$, $\delta_e = 30^0$.

Let $y = [y_1, y_2]^T = [q_1, q_2]^T$, $u = [u_1, u_2]^T$, $x = [q_1, \dot{q}_1, q_2, \dot{q}_2]^T$ and

$$\begin{aligned}
F(x) &= \begin{bmatrix}f_1(x)\\ f_2(x)\end{bmatrix}\\
&= -M^{-1}\begin{bmatrix}-h\dot{q}_2 & -h(\dot{q}_1+\dot{q}_2)\\ h\dot{q}_1 & 0\end{bmatrix}\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\end{bmatrix},\\
G(x) &= \begin{bmatrix}g_{11}(x) & g_{12}(x)\\ g_{21}(x) & g_{22}(x)\end{bmatrix} = M^{-1}
\end{aligned}$$

then, the robot system can be expressed as follows

$$\ddot{y} = F(x) + G(x)u$$

which is in the input–output form given by (2). Since the matrix M is positive definite, then $G(x) = M^{-1}$ is positive definite. The control objective is to force the system outputs q_1 and q_2 to track the sinusoidal desired trajectories $y_{m1}(t) = \sin(t)$ and $y_{m2}(t) = \sin(t)$, respectively. Within this simulation, the nonlinear functions $F(x)$ and $G(x)$ are assumed completely unknown, two fuzzy systems in the form of (11) are used to approximate the elements of $F(x)$, and three are used to approximate the elements of $G(x)$. The fuzzy systems used to describe $F(x)$ have $q_1(t)$, $\dot{q}_1(t)$, $q_2(t)$ and $\dot{q}_2(t)$ as inputs, and the ones used to describe $G(x)$ have $q_1(t)$ and $q_2(t)$ as inputs.

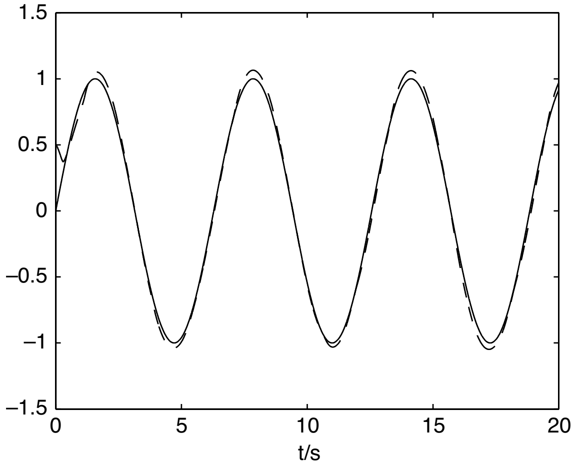

Fig. 1. Position tracking curves of link 1: $y_{m1}(-)$, $y_1(--)$.

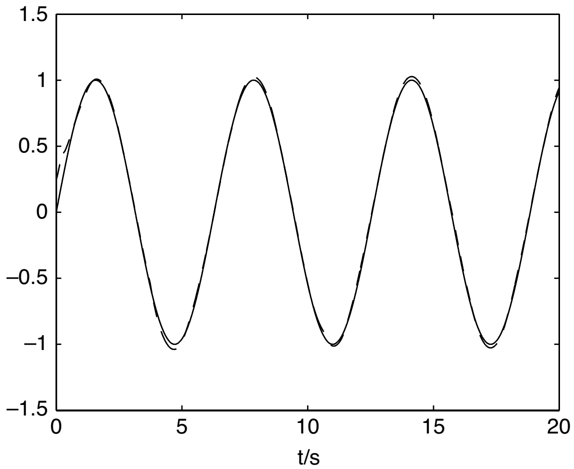

Fig. 2. Position tracking curves of link 2: $y_{m2}(-)$, $y_2(--)$.

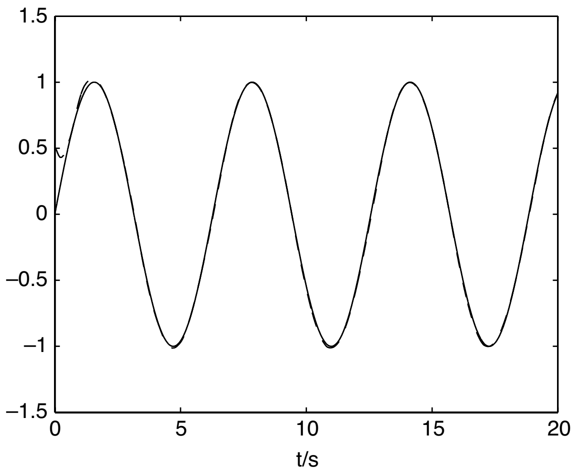

Fig. 3. Position tracking curves of link 1: $y_{m1}(-)$, $y_1(--)$.

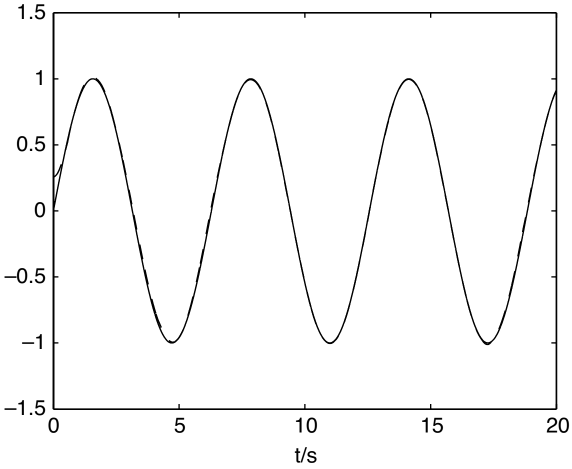

Fig. 4. Position tracking curves of link 2: $y_{m2}(-)$, $y_2(--)$.

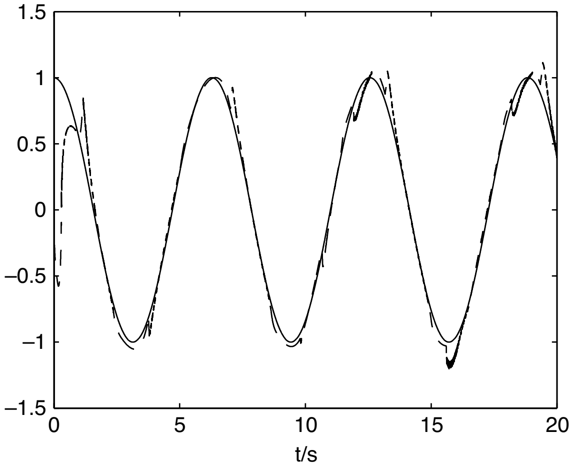

Fig. 5. Velocity tracking curves of link 1: $\dot{y}_{m1}(-)$, $\dot{y}_1(--)$.

In the simulation, the fuzzy membership functions are defined for every variable $q_1, \dot{q}_1, q_2, \dot{q}_2$ as follows [11]:

$$\mu_{F_i^1}(x_i) = \exp\left(-\frac{1}{2}\left(\frac{x_i + 1.25}{0.6}\right)^2\right),$$

$$\mu_{F_i^2}(x_i) = \exp\left(-\frac{1}{2}\left(\frac{x_i}{0.6}\right)\right),$$

$$\mu_{F_i^3}(x_i) = \exp\left(-\frac{1}{2}\left(\frac{x_i - 1.25}{0.6}\right)^2\right).$$

Let the initial conditions be $x(0) = [0.5, 0, 0.25, 0]^T$, and each element of $\theta_{f1}(0), \theta_{f2}(0), \theta_{g11}(0), \theta_{g12}(0)$ and $\theta_{g22}(0)$ are all chosen randomly in the interval $[-0.5, 0.5]$.

The design parameters are chosen as $k_{01} = 4$, $k_{02} = 4$, $\gamma_f = 0.5$, $\gamma_g = 0.5$, $\varepsilon_1 = 0.1$, $\varepsilon_2 = 0.1$, $\lambda_1 = 4$, $\lambda_2 = 4$.

In order to highlight the effectiveness of the proposed control scheme, we simulate the control design with the designed parameter $\beta = 0.05$ and $\beta = 0.5$, the position tracking results for the first link and the second link with $\beta = 0.05$ are shown in Figs. 1 and 2, respectively, and the least mean square error of $e_1(t)$ is 0.0033, and $e_2(t)$ is 0.0012, those for the first link and the second link with $\beta = 0.5$ are shown in Figs. 3 and 4, respectively, and the least mean square error of $e_1(t)$ is 0.0018, and $e_2(t)$ is 0.0005. The velocity tracking results for the first link and the second link with $\beta = 0.05$ are shown in Figs. 5 and 6, respectively, and the least mean square error of $\dot{e}_1(t)$ is 0.0397, and $\dot{e}_2(t)$ is 0.0089, those for the first link and the second link with $\beta = 0.5$ are shown in Figs. 7 and 8, respectively, and the least mean square error of $\dot{e}_1(t)$ is 0.0267, and $\dot{e}_2(t)$ is 0.0078.

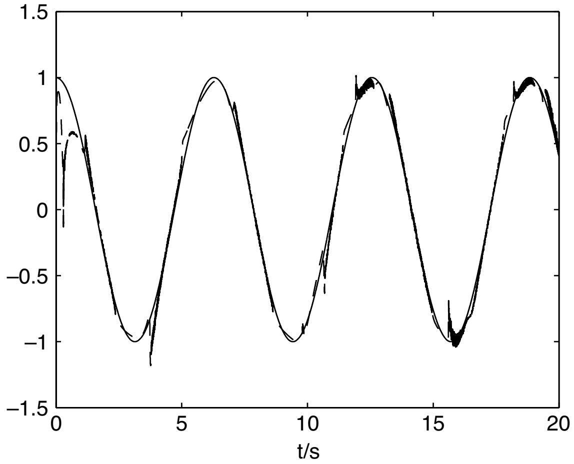

Fig. 6. Velocity tracking curves of link 2: $\dot{y}_{m2}(-)$, $\dot{y}_2(--)$.

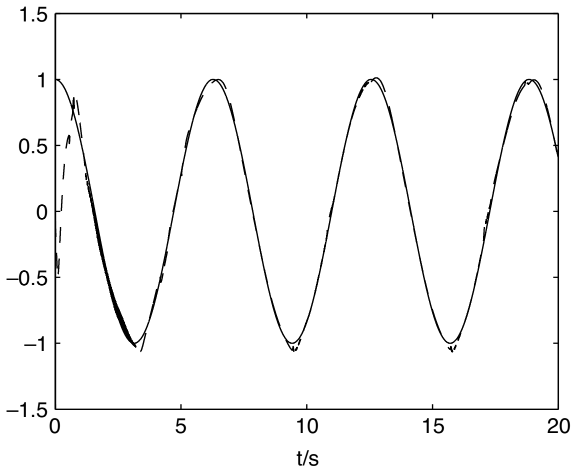

Fig. 7. Velocity tracking curves of link 1: $\dot{y}_{m1}(-)$, $\dot{y}_1(--)$.

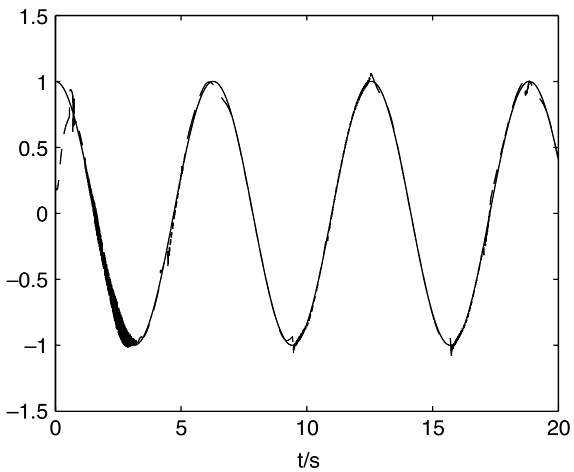

Fig. 8. Velocity tracking curves of link 2: $\dot{y}_{m2}(-)$, $\dot{y}_2(--)$.

From these simulation results, it is obvious that the tracking errors can be reduced by increasing the value of designed parameter β, and the tracking capability of the proposed control scheme is quite satisfactory.

5. Conclusions

In this paper, an indirect adaptive fuzzy control scheme is developed for a class of uncertain MIMO nonlinear systems. Within this scheme, the fuzzy logic systems are used to approximate the plant's unknown dynamics. By using the symmetric matrix decomposition technique to avoid the possible controller singularity problem. The proposed design scheme guarantees that all the signals in the resulting closed-loop system are UUB. Moreover, the tracking errors can be made small enough if the designed parameter β is chosen to be sufficiently large.

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