Simulation of Correlated Wind Speed Data for Economic Dispatch Evaluation

Daniel Villanueva, Andrés Feijóo, and José Luis Pazos

***Abstract*—The Economic Dispatch problem consists of minimizing the cost of producing the power demanded by an electrical power system, by means of the suitable dispatching of the power production between the available generators. The difficulty in predicting wind power generation means that penalty and reserve costs must be considered when it is included in the evaluation. Analyzing the output power of each wind turbine individually is not enough when evaluating these costs and the correlation between wind speed values must be considered as another input because it also has an influence. This paper introduces a new method for generating correlated wind power values and explains how to apply the method when evaluating Economic Dispatch. A case study is provided to analyze whether considering correlation in the problem has any influence or not.**

***Index Terms*—Correlation, economic dispatch, Monte Carlo simulation, Weibull distributions, wind power.**

I. Introduction

ECONOMIC dispatch (ED) consists of dispatching the power to be generated among available generators, in order to obtain the most efficient, low-cost, and reliable operation of a power system. It considers operating limits, availability, and reliability, and minimizes costs so that both load demand and losses are supplied [1]. It, therefore, plays a key role in power system planning and operation.

As wind power increases its share ratio in electrical networks, the ED problem must consider scheduled wind power and its costs. Even though wind power forecasting methodologies [2] have been considerably improved during the last years, wind power cannot be scheduled with total accuracy.

The generally used models for wind power forecasting are based on several factors: current data and atmospheric behavior, such as the numerical weather prediction (NWP); historical data, such as auto regressive (AR), auto regressive moving average (ARMA), etc.; spatial correlation; artificial intelligence; or a combination of these [3]–[6].

Therefore, taking into account that scheduled and available wind power may not coincide, the costs to be considered in the ED problem are different in nature, as will be explained later.

The ED problem is solved *a priori*, which means that the power to be produced is scheduled. Flexible plants can set output power to the required value, so both the scheduled and the produced power values coincide unless the unit is malfunctioning. There are other things to consider when wind power is taken into account because the available and scheduled powers at the corresponding wind turbine (WT) may differ and give rise to costs that must be evaluated.

Several types of procedures can be applied to solve the ED problem. Analytical solutions like those in [7] are very difficult to apply when considering wind power correlation so numerical solutions like the ones in [8] are preferred. Therefore, the ED evaluation carried out in this paper is based on four terms that depend on the following:

1) the scheduled power of flexible plants;
2) the scheduled wind power;
3) the difference between available and scheduled wind power, when it is positive;
4) the difference between scheduled and available wind power, when it is positive.

From the point of view of the system operator, there is a first term for the cost due to the sum of the power provided by the conventional generators.

The second one is due to the amount to be paid to wind power producers according to agreements established with them.

A term is included that evaluates the cost of the difference between available and scheduled wind power. These costs include the payment to the wind power producers for not using the available power, which is wasted or diverted to another generation facility like a hydro pumping station. Therefore, the third term is basically related to penalty costs due to not using all the available power in the network, i.e., it evaluates the cost of under-forecasting wind power.

On the other hand, if the wind power is over-forecasted, the power requested by the load demand has to be supplied anyway, so the power must be purchased from an alternative source. Therefore, the cost of the reserve power or the cost of the power purchased through an interconnection is evaluated in the fourth term.

So, these four terms evaluate the costs in order to solve the ED problem. However, this is a general model to evaluate ED and it is adaptable to all possible situations, so any term can be removed depending on specific cases.

Notice that if the total power generated and demanded in an electrical network were not equal, then steady-state security could be affected, because this balance is needed for keeping adequate operating conditions. Thus, the electrical network could be seriously influenced by under- or over-forecasting wind power if the steps towards achieving the balance were not taken into account.

Manuscript received January 26, 2011; revised July 22, 2011; accepted August 12, 2011. Date of current version December 16, 2011. This work was supported by the autonomous government Xunta de Galicia, under Contract 08REM009303PR.

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Digital Object Identifier 10.1109/TSTE.2011.2165861

As explained in [8], the ED problem can be solved by taking into account the probabilistic nature of the wind speed: its statistical distribution can be approximated using a Weibull distribution and the probability density function (pdf) of the wind power can be obtained. However, the correlation between wind speed values and, consequently, between wind power values has to be included in the ED problem.

Generally, it can be said that correlation between wind speed values has great influence in electrical networks with WTs. In this paper, a method is derived to simulate series of WT output power data based on given correlated wind speed values between the locations where they are installed and also on lag-one autocorrelated values for each one. The series of WT output power data represent all possible situations. This method upgrades former approaches to this simulation in accuracy and computation time [9]–[12]. The WT output power series are used to obtain a more realistic result in the minimization of total cost, now defined on the basis of these data series.

Notice that the method proposed in this paper does not correspond to a wind power forecasting methodology but is instead a way to define all possible wind power generation states in a group of WTs.

The Interior Point Method is applied to solve the minimization problem, as it is usually used as an optimization tool for ED [13]–[16].

This paper is organized as follows: Section II explains how to simulate series of correlated and autocorrelated wind speed values; Section III outlines how to convert these wind speed series into WT output power series; Section IV describes the ED problem considering wind power and how to introduce the correlation factor; Section V shows a case study and Section VI states the conclusions.

II. Wind Speed Series

First, let us briefly comment on some of the former approaches proposed to obtain correlated wind speeds. The objective is to find a number of wind speed series where each one fulfills a given Weibull distribution, with parameters (λ, k), and the correlation coefficients are those provided by the correlation matrix.

In [9], a method that keeps the Weibull features of each of the series and obtains the exact correlation matrix is applied. However, in this method Spearman rank correlations are considered [17], so this feature must be considered when data and results are used.

Evolutionary algorithms have been used in [10] for the same purpose. They are based on the initial generation of wind speed series fulfilling the distribution features, followed by the rearranging of the values in each of the series in a trend towards obtaining the desired correlation matrix. Although the results provided by this method are very accurate, the computation time increases considerably as the number of locations rises.

In [11], a method has been proposed that is based on the sum of two squared Normal distributed variables in order to obtain the square of a Weibull distributed one, which involves solving equations by means of iterative processes. This method has nothing to do with the one proposed in this paper.

A minimization process is proposed in [12], thus using the decomposition of Weibull distribution variables as weighted sum of Uniform ones.

Except for the method proposed in [9], the time consumed by these methods tends to be increasingly greater as the number of locations increases, due to computational issues. Moreover, the accuracy of these methods depends on the error accepted by the minimization procedures. The method proposed in this paper reduces the computational time to a minimum because it does not use iteration processes, as it holds the starting correlation values, is fully accurate, and also considers parametric correlation, which is the convention.

In the methods mentioned above, only the correlation between wind speed data has been considered. However, in [11], the autocorrelation for each location has also been computed. In this paper, both types of correlation are taken into account.

A. Conversion of Normal Distributed Series Into Weibull Distributed Ones

The wind speed pdf at a certain location can be described by a Weibull distribution [18]–[20].

Widely used in statistics, the cumulative distribution function (cdf) of a continuous variable with a given distribution makes a transformation possible between this distribution and a Uniform one. This feature is usually used in reverse to simulate random data, when the cdf is inverted to generate uniformly distributed data between 0 and 1 so that random data can finally be created with the desired cdf.

For example, in the case of a Weibull distributed variable u, the variable F_u, obtained according to (1) is uniformly distributed

$$F_u = 1 - \exp\left(-\left(\frac{u}{\lambda}\right)^k\right) \tag{1}$$

where λ is the scale parameter and k the shape parameter of the Weibull distribution [21].

Exactly the same can be said for Normal distributions, as can be seen as follows:

$$F_x = \frac{1}{2}\left(1 + \operatorname{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right) \tag{2}$$

where F_x is the cdf of the variable x, erf( ) is the error function, defined in (3), and μ and σ are the mean and the standard deviation of the Normal distribution.

The error function, erf( ), is continuous, and its features are shown in many handbooks of mathematics [22]

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2)dt. \tag{3}$$

The reason for using the feature mentioned above is because it is intended to operate a conversion from Normal distributed data into Weibull distributions. First, the value obtained from the Normal distribution is converted into a value belonging to a Uniform one, and then this value is converted into a new one, corresponding to a Weibull distribution. So the operations involve beginning with Normal distributed data and using (2), in a direct way, and (1), in an inverse way, to obtain the Weibull

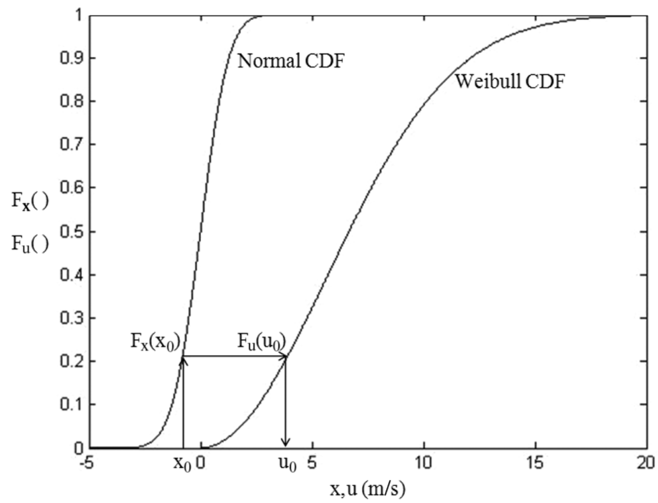

Fig. 1. Process to convert Normal values into Weibull ones.

distributed data. The whole process can be performed by applying (4) to a Normal distributed variable x, with parameters $\mu = 0$ and $\sigma = 1$, obtaining a Weibull distribution one u with parameters λ and k

$$u = \lambda \left(-\log \left(\left(1 - \mathrm{erf}\left(\frac{x}{\sqrt{2}} \right) \right) \Big/ 2 \right) \right)^{\frac{1}{k}} \tag{4}$$

where log denotes natural logarithm. The use of the Standard Normal distribution will be justified later.

Fig. 1 shows the process to convert a Normal value (x_0) into a Weibull one (u_0) by using (4).

For example, starting from a Normal value of $x_0 = -0.8$, the corresponding value of the Normal cdf is $F_x(x_0) = 0.2119$, this value is taken as the Weibull cdf, $F_u(u_0)$, which provides a value of $u_0 = 3.9034$.

Therefore, instead of working with Weibull distributions to obtain correlated and autocorrelated series of data, it is easier to generate the data using Standard Normal distributions, which are converted into Weibull ones in a second step, as explained earlier.

In order to check that the correlation coefficients keep their value once the change suggested in (4) is applied, a Monte Carlo simulation has been performed. The correlation coefficients of two Bivariate distributions have been compared: the Standard Normal and the Weibull one. Taking values of the Standard Normal correlation coefficient from -1 to 1, and Weibull parameters equal to $\lambda_1 = 7$, $k_1 = 2.2$, $\lambda_2 = 9$, and $k_2 = 1.9$, the relationships between both correlation coefficients are shown in Fig. 2.

The Monte Carlo simulation has been repeated for Weibull parameters varying in the following intervals: $\lambda = [1, 15]$ and $k = [1, 3]$ in steps of 0.1 for both. However, for any combination of parameters in the intervals, the graph is exactly the same.

Therefore, it can be concluded that the correlation coefficients are eventually the same.

B. Simulation of Autocorrelated Series

In order to simulate wind speed series with known parameters, two types of correlation have to be considered. First, the temporal correlation, which takes into account the dependence between a sample value and the previous ones. When this dependence is considered among samples from the same location it is called autocorrelation. Second, the spatial correlation considers the dependence between simultaneous sample values at a pair of locations. The former is developed in this subsection and the latter in the following one.

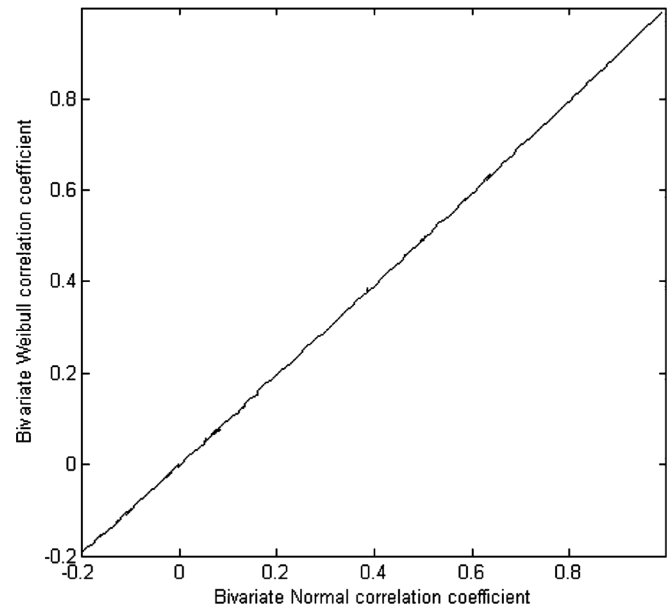

Fig. 2. Comparison between correlation coefficients.

The simplest way to take temporal correlation into account is to consider the lag-one autocorrelation for each location, which reduces this type of correlation to the dependence between each single value and the previous one from the same location [23]. In this case, a first order autoregressive process, AR(1), can be applied [11].

An AR(1) can be operated according to (5)

$$x_t = \phi x_{t-1} + a_t \tag{5}$$

where x_t is the value of the variable x at time t, x_{t-1} is its value at $t-1$, ϕ is a constant value, the lag-one autocorrelation value, and a_t is a normal distributed variable, with parameters $\mu_a = 0$, and σ_a.

The lag-one autocorrelation value, ϕ, for a certain location is calculated according to the following steps:

1) to take the series of available wind speed values at that location;
2) to move the values of this series one step backwards;
3) to obtain the correlation coefficient between both series.

The values of standard deviations σ_x and σ_a are related according to

$$\sigma_x^2 = \frac{\sigma_a^2}{1 - \phi^2}. \tag{6}$$

Taking into account that x follows a Standard Normal distribution, i.e., $\sigma_x = 1$, the value of σ_a is obtained as in

$$\sigma_a = \sqrt{1 - \phi^2}. \tag{7}$$

Therefore, in order to simulate series of autocorrelated values according to an AR(1) model for a Standard Normal distribution variable, with lag-one autocorrelation value ϕ, the process is as follows:

1) to obtain σ_a from (7);
2) to give an initial value x_0;
3) to obtain a random value a_1, taking into account that $a_t \sim N(0, \sigma_a)$;
4) to generate x_1 according to (5);
5) to repeat steps iii) and iv) as many times as needed.

This process has to be performed independently for each location considered, due to the different values of the lag-one autocorrelation.

C. Obtaining Correlated Series

The spatial correlation considers the dependence between simultaneous values at a pair of locations. Therefore, when a group of locations are involved, the correlation coefficients for each pair are organized in a matrix called correlation matrix.

As the variables are Standard Normal distributed, the Cholesky decomposition is used in order to obtain correlated series. It is based on obtaining a lower triangular matrix L, that fulfils (8), where Ω_y is the desired correlation matrix.

The desired correlation values between a pair of locations is usually obtained from existing series of wind speed data, but in cases when some of the correlation values are not available, an approximation based on the distance between both locations can be used [24]

$$\Omega_y = LL^t \tag{8}$$

where Ω_y, L, and L^t are $m \times m$ matrices, where m is the number of variables, and L^t means transposition of L.

If L is applied to a vector of uncorrelated samples X, the resulting vector Y contains samples with the correlations provided by Ω_y

$$Y = LX. \tag{9}$$

The correlated series of Standard Normal distributed data can be converted easily into correlated wind speed series that strictly follow the Weibull distribution for each location and keep the correlation provided by Ωy, using (4). The process also keeps the lag-one autocorrelation introduced before the Cholesky decomposition is applied, i.e., the vector X in (9) is uncorrelated but each sample of values is autocorrelated.

Notice that if the Cholesky decomposition were applied to variables with a distribution different from the Normal one, the results obtained in Y would not follow a known distribution. If each series in X follows a Standard Normal distribution $(\mu = 0, \sigma = 1)$, each series in Y will follow the same one, as proven in the Appendix. Moreover, as the Cholesky decomposition performs linear combination of the series in X to obtain the series in Y and as can be deduced from the Central Limit Theorem, any type of distribution will tend to Normal ones when it is applied; therefore, there is no other type of distribution to be used instead in this method.

The same method can be proposed but using Uniform distributions instead of Normal ones. In this case, the Uniform features in X will be lost after applying (9) and these series will tend to Normal ones as long as their number increases. Therefore, in this case, the change of variables from Uniform to Weibull will not provide the desired series of values because they do not fulfill the specifications required. The change of variables will try to convert Uniform series of values into Weibull ones, but the data in Y will not be series of Uniform values; therefore, the result will not have the required distribution.

Obviously, if Weibull distributed series of data are used directly in the method, the results obtained do not provide the desired requirements, i.e., after applying (9) the series will not fulfill the Weibull condition.

D. Wind Speed Series

The ED is carried out some time before the real situation occurs. So, in order to obtain series of wind speed data that can be considered as groups of possible situations, the autocorrelation has to be taken into account to obtain one group of possible wind speed values, using as many periods as there will be between making the calculations and applying them. Finally, the correlation has to be considered between all groups.

For example, if the wind speed data are taken every 10 min, and the ED is going to be calculated 24 hours in advance, as happens in the Spanish market, then the procedure consists of calculating autocorrelated series of 144 wind speed data. Starting from a group of known samples, corresponding to the current values, the group of samples formed by the 144th values can be described as a feasible group of values. Then, in order to obtain several groups of this type, the procedure is repeated 10 000 times. Afterwards, the method to obtain the correlation between all groups of values is applied. So, 10 000 correlated groups of wind speed values, considering autocorrelation, are provided.

III. Wind Power Series

The next step is to obtain wind power series from these wind speed series. This is usually performed by using the power curve, which is a group of pairs of data points, provided by the WT manufacturer. However, in order to make it computationally effective, a cubic spline interpolation is used. This method fits a different polynomial for each interval, which is defined by two pairs of data points. So the power curve can be modeled as a piecewise polynomial curve, as shown in the following:

$$\omega(u) = \alpha_j u^3 + \beta_j u^2 + \gamma_j u + \delta_j \quad u_j \leq u < u_{j+1} \tag{10}$$

where u is the wind speed and ω is the wind power, u_j and u_{j+1} are the wind speed data provided by the manufacturer, α_j, β_j, γ_j, and δ_j are the polynomial coefficients corresponding to the jth interval.

The values given by this model have been compared with the manufacturer data of a Vestas V100 WT [25] and the result of this comparison can be seen in Fig. 3.

Therefore, the wind power series for each location are obtained simply by applying this model to the data provided by the manufacturer of the WT installed there.

Finally, simulated series of wind power values provided by WTs are obtained by taking wind speed correlation and autocorrelation into account.

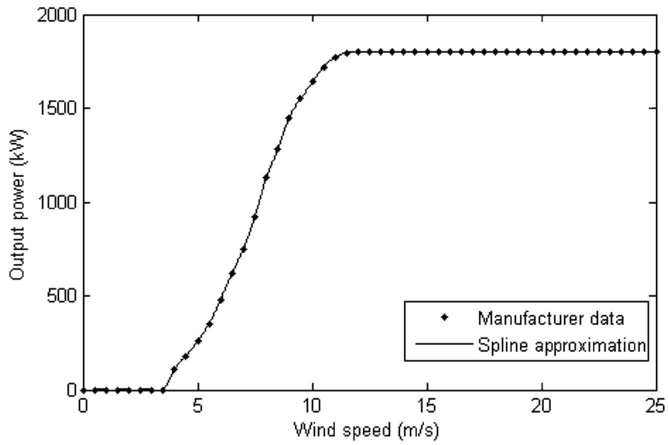

Fig. 3. Spline approximation of the power curve of a VESTAS V100 WT.

IV. Economic Dispatch

A. Economic Dispatch Including Wind Power

The ED can be mathematically expressed as an optimization problem, where the objective is to minimize the total cost of generating the requested power, considering the power values scheduled in all generators for a certain time period, when the ED is intended to be solved.

If the power sources of the electrical network can be fully programmable, as happens with flexible plants, then generation coincides with the scheduled load demand. However, if the electrical network includes WTs, the scheduled and available power might not coincide, because the primary source, the wind, can neither be controlled, nor forecasted with full accuracy.

Therefore, when WTs are involved the cost minimization problem includes more terms than usual because the available wind power may differ from the scheduled one.

First, it is necessary to consider what happens when not all the available wind power is used, i.e., when there is more available wind power than the scheduled one. In this case, a penalty cost is applied, which is the payment to the wind power producer for that additional power.

Second, when the scheduled wind power is not achieved, or in other words, when the available wind power is lower than the expected one, a reserve power source has to provide the difference. A reserve cost has to be taken into account when there are WTs in the electrical network.

On the other hand, it can be said that the load demand can be established with high accuracy for a certain period of time, based on historical data. However, it must be pointed out that there is always a degree of uncertainty in its value because, in fact, it is forecasted in some way. In the method proposed in this paper, the load demand is considered as known.

Moreover, wind power and load demand are both forecasted and in some cases, due to the influence of the sun, both follow similar paths throughout the day. However, this cannot be stated as a general rule and so, in the method proposed here, load demand and wind power are considered statistically independent without a significant error.

So, the ED problem consists of obtaining values for all p_i, power scheduled for the ith conventional generator (CG), and ω_i, power scheduled for the ith WT, pursuing to minimize

$$g = \sum_{i=1}^{n_{\rm cg}} C_i^{\rm cg}(p_i) + \sum_{i=1}^{n_{\rm wt}} C_i^{\rm wt}(\omega_i) + \sum_{i=1}^{n_{\rm wt}} C_i^{\rm wp}\left(\omega_i^{\rm av} - \omega_i\right) + \sum_{i=1}^{n_{\rm wt}} C_i^{\rm wr}\left(\omega_i - \omega_i^{\rm av}\right) \quad (11)$$

where $n_{\rm cg}$ is the number of CGs and $n_{\rm wt}$ is the number of WTs. Constraints described by (12)–(14) also need to be fulfilled

$$\sum_{i=1}^{n_{\rm cg}} p_i + \sum_{i=1}^{n_{\rm wt}} \omega_i = D \quad (12)$$

$$p_i^{\min} \le p_i \le p_i^{\max} \quad 1 \le i \le n_{\rm cg} \quad (13)$$

$$0 \le \omega_i \le \omega_i^r \quad 1 \le i \le n_{\rm wt} \quad (14)$$

where D is the total load demand plus losses, $p_i^{\min}$ and $p_i^{\max}$ are the minimum and maximum powers that the ith generator can supply, and ω_i^r is the rated power of the ith WT. In the following paragraphs, the terms of (11) are explained more in detail.

In (11), $C_i^{\rm cg}()$ is the cost function for the ith CG, and it is usually expressed as in

$$C_i^{\rm cg}(p_i) = a_i p_i^2 + b_i p_i + c_i \quad (15)$$

where a_i, b_i, and c_i are parameters for the ith CG.

On the other hand, the cost function for the ith WT, $C_i^{\rm wt}()$ can be expressed as in

$$C_i^{\rm wt}(\omega_i) = d_i \omega_i \quad (16)$$

where d_i is a parameter for the ith WT.

The penalty cost $C_i^{\rm wp}()$, which is a function of the difference between the available and scheduled wind power, is considered proportional to the expected value of that difference $E()$

$$C_i^{\rm wp}\left(\omega_i^{\rm av} - \omega_i\right) = k_{p_i} E(\omega - \omega_i) \quad \omega > \omega_i \quad (17)$$

where k_{pi} is a constant value for the ith WT, and generally depends on local regulations.

The expected value is calculated as in

$$E(\omega - \omega_i) = \int_{\omega_i}^{\omega_i^r} (\omega - \omega_i) f(\omega) d\omega \quad \omega > \omega_i \quad (18)$$

where ω is the variable, ω_i is the scheduled power for the ith WT and $f(\omega)$ is the pdf of the wind power.

The reserve cost $C_i^{\rm wr}()$ is considered as a function of the difference between the scheduled and the available wind power, and is taken as proportional to the expected value of that difference

$$C_i^{\rm wr}\left(\omega_i - \omega_i^{\rm av}\right) = k_{r_i} E(\omega_i - \omega) \quad \omega < \omega_i \quad (19)$$

where k_{ri} is a constant value for the ith WT, and depends on the agreement between seller and buyer and on local regulations.

Equation (20) shows how to obtain the expected value $E()$ as follows:

$$E(\omega_i - \omega) = \int_{0}^{\omega_i} (\omega_i - \omega) f(\omega) d\omega \quad \omega < \omega_i. \quad (20)$$

B. Economic Dispatch Considering Correlated Wind Power

The consideration of correlation between wind power values affects the third and fourth terms of (11), which are evaluated as z in

$$z=\sum_{i=1}^{n_{\rm wt}} C_i^{\rm wp}\left(\omega_i^{\rm av}-\omega_i\right)+\sum_{i=1}^{n_{\rm wt}} C_i^{\rm wr}\left(\omega_i-\omega_i^{\rm av}\right). \quad (21)$$

According to [8], both types of functions can be expressed as proportional to the expected value of the differences, as in

$$z=\sum_{i=1}^{n_{\rm wt}} k_{p_i}E\left(\omega_i^{\rm av}-\omega_i\right)+\sum_{i=1}^{n_{\rm wt}} k_{r_i}E\left(\omega_i-\omega_i^{\rm av}\right). \quad (22)$$

Applying the properties of linearity of the expectation, (22) can be easily converted into

$$z=\sum_{i=1}^{n_{\rm wt}} E\left[k_{p_i}\left(\omega_i^{\rm av}-\omega_i\right)\right]+\sum_{i=1}^{n_{\rm wt}} E\left[k_{r_i}\left(\omega_i-\omega_i^{\rm av}\right)\right]. \quad (23)$$

Again, (23) turns into (24) due to linearity, as follows:

$$z=E\left[\sum_{i=1}^{n_{\rm wt}} k_{p_i}\left(\omega_i^{\rm av}-\omega_i\right)\right]+E\left[\sum_{i=1}^{n_{\rm wt}} k_{r_i}\left(\omega_i-\omega_i^{\rm av}\right)\right]. \quad (24)$$

And (25) is obtained due to the linearity of expectation, as follows:

$$z=E\left[\sum_{i=1}^{n_{\rm wt}} k_{p_i}\left(\omega_i^{\rm av}-\omega_i\right)+\sum_{i=1}^{n_{\rm wt}} k_{r_i}\left(\omega_i-\omega_i^{\rm av}\right)\right]. \quad (25)$$

Therefore, considering series of values instead of a function, the value of z is obtained by evaluating the mean of all possible values of z, each named z_i, as in

$$z=\frac{\sum_{j=1}^{n} z_j}{n} =\frac{\sum_{j=1}^{n} E\left[\sum_{i=1}^{n_{\rm wt}} k_{p_i}\left(\omega_i^{{\rm av}_j}-\omega_i\right)+\sum_{i=1}^{n_{\rm wt}} k_{r_i}\left(\omega_i-\omega_i^{{\rm av}_j}\right)\right]}{n} \quad (26)$$

where n is the number of elements in the series of wind power. Each z_j is calculated according to (25), where $\omega_i^{{\rm av}j}$ is the wind power available in the ith WT for the jth simulation value and where ω_i is the scheduled wind power in the ith WT, which is the same for the n simulation values.

C. Optimization Technique for the Economic Dispatch

The techniques that have been usually applied to solve the ED are based on the Lagrangian Relaxation [26], Direct Search Method [27], Evolution Programming [28], Particle Swarm Optimization [29], Genetic Algorithms [30], Simulated Annealing [31], and Interior Point Method.

In this paper, the Primal-Dual Interior Point Method has been applied to solve the ED, which is already implemented in most of the mathematical packages and is independent of the problem being solved. The application of the Primal-Dual Interior Point Method is described in [13]–[16].

TABLE I
PARAMETERS OF THE CONVENTIONAL UNITS

		Unit 1	Unit 2
Minimum output power (MW)	$P_{\min}$	5	5
Maximum output power (MW)	$P_{\max}$	40	30
Cost parameters	a (€/MW^2)	25	25
	b (€/MW)	20	25
	c (€)	25	20

TABLE II
COST PARAMETERS OF THE WTs

	1	2	3	4	5	6	7	8
d (€/MW)	30	25	35	25	30	25	35	25
k_p (€/MW)	160	160	160	160	160	160	160	160
k_r (€/MW)	200	200	200	200	200	200	200	200

TABLE III
WIND SPEED PARAMETERS

	1	2	3	4	5	6	7	8
λ	8.13	8.24	7.52	9.24	8.11	7.24	7.36	9.09
k	1.99	2.30	2.11	2.41	1.79	2.12	2.03	2.14
ϕ	0.7	0.8	0.6	0.7	0.8	0.7	0.6	0.8

V. CASE STUDY

This section compares two methods for evaluating ED, the one proposed in [8] and the one proposed in this paper. An electrical network with two conventional units and eight WTs is used in this case study, where different load values for the system are evaluated. Both conventional units are required to be operating for any load value. The parameters regarding these units are provided in Table I.

The cost parameters of the WTs are listed in Table II, and the wind speed parameters of the corresponding locations are shown in Table III.

In Table III, λ and k are the parameters of the Weibull distribution of the wind speed for each location, and ϕ is the lag-one autocorrelation value.

There are four Vestas V80-2.0 MW WTs and four Vestas V90-3.0MW.

In order to check the differences between both methods, just two cases have been taken into account. The first one is for no correlation, where correlations have not been considered in the analysis, and the second one is for high correlation, where a correlation value of 0.9 is utilized for all pairs to obtain the correlated wind speed values. These two cases are computed for different values of total load, with values from 10 to 35 MW.

The ED minimization problem is solved using the Interior Point algorithm providing the results shown in Fig. 4. The differences can be seen between the method proposed here and the one proposed in [8], both for total costs and for wind power generation costs.

For example, for a total load of 15 MW, the consideration of correlation provides a cost of 1665 €/h, whereas if it is not taken into account, the cost is 1259 €/h which means that if correlation is not taken into account the error produced when evaluating the optimal cost is around 25%. The same happens for a total load

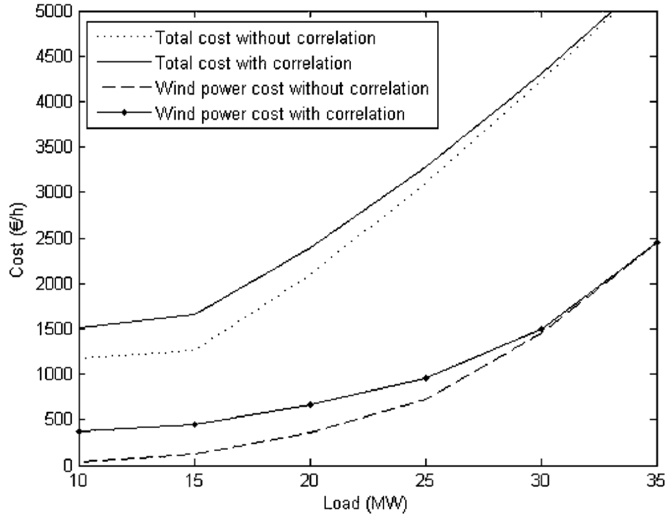

Fig. 4. Costs comparison, neglecting and including correlation.

of 25 MW, where the result is 3288 €/h against 3108 €/h, respectively, although in this case the error is around 5%. As long as the total load is increased, the influence of considering the correlation, in this case, is reduced. For a total load of 35 MW the difference is just 22 €/h, so there is almost no influence of the correlation in this case.

It can be said that in this case study the amount of error when evaluating the optimal cost of generating the load demanded is higher as the load demand decreases. The low limit for that situation is 10 MW because it is the sum of the minimum power generated by conventional generators, and so in this electrical system it is not possible to generate less than 10 MW.

Fig. 4 also shows the comparison of the wind power cost evaluated with and without correlation. In this case, for each value of load demand, the difference between both methods is almost the same as in the case of the total cost, which means that the cost of the power injected by conventional generators in the electrical network is the same when utilizing both methods.

VI. Conclusion

In this paper, a new method has been described for including correlation and autocorrelation in series of randomly generated wind speed distributions, keeping their distribution features (i.e., Weibull parameters). It makes use of a relationship established between the cdfs of Normal and Weibull distributions.

The method has been applied to the generation of correlated wind speed values to obtain wind power values that contribute to solving the ED problem. The introduction of correlation and autocorrelation of wind speed series enables a more realistic approach to the problem itself.

The proposed model for the ED analysis is to some extent more complex than the previous one. However, any increase in computing time due to upgrading the method by including the simulation of correlated wind speed values is not appreciable.

Obviously, the influence of wind power correlation in the total cost of generating the demanded power becomes more considerable as the percentage of wind power becomes higher in the total power produced by the electrical network. The percentage of wind power depends on the load being demanded but also on network configuration. So, if the number of WTs increases with respect to conventional units, the influence will be greater.

The results of the case study reveal that it is relevant to consider correlation in the analysis. In fact, in our example, the results give us a more expensive generation cost when correlation is included. The correct interpretation of the results is that when correlations are not included in the analysis an error is being produced in the generation cost estimation. In the case of our example, we can conclude that these costs are underestimated when correlations are not taken into account.

Appendix
Cholesky Decomposition

The decomposition of a Hermitian, positive-definite matrix into the product of a lower triangular matrix and its conjugate transpose is called the Cholesky decomposition.

The Cholesky decomposition is mainly used for the numerical solution of linear equations, linear least square problems, nonlinear optimization, or Kalman filters. In this paper, it has been used in the MC simulation.

Given a matrix of uncorrelated series of samples X, and the lower triangular matrix L, obtained from the Cholesky decomposition of the desired correlation matrix Ω_y, a matrix Y can be obtained of correlated series of samples according to Ω_y by means of

$$\begin{pmatrix} y_{11} & y_{12} & \cdots & y_{1n} \\ y_{21} & y_{22} & \cdots & y_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ y_{m1} & y_{m2} & \cdots & y_{mn} \end{pmatrix} = \begin{pmatrix} l_{11} & 0 & \cdots & 0 \\ l_{21} & l_{22} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ l_{m1} & l_{m2} & \cdots & l_{mm} \end{pmatrix} \cdot \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{pmatrix} \tag{A.1}$$

where n is the number of samples of each series and m is the number of series.

Moreover, if each sample series follows a Normal distribution, the results are also sample series that follow a Normal distribution, which can easily be demonstrated.

The jth series is formed by n elements of the type expressed in

$$y_{ji} = \sum_{l=1}^{j} l_{jl} \cdot x_{li} \quad 1 \leq i \leq n. \tag{A.2}$$

If x_{li} is distributed according to a $N(\mu_l, \sigma_l)$, then y_{ji} will tend to follow

$$N\left(\sum_{l=l}^{j} l_{jl} \cdot \mu_l, \sqrt{\sum_{l=l}^{j} l_{jl}^2 \cdot \sigma_l^2}\right). \tag{A.3}$$

Moreover, if $\mu_1 = \mu_2 = \ldots = \mu_l = \mu$ and $\sigma_1 = \sigma_2 = \ldots = \sigma_l = \sigma$, then y_{ji} will follow

$$N\left(\left(\sum_{l=l}^{j} l_{jl}\right)\mu, \sqrt{\sum_{l=l}^{j} l_{jl}^2 \sigma}\right). \tag{A.4}$$

And, in the Standard case, if $\mu = 0$ and $\sigma = 1$, then y_{ji} will be distributed according to

$$N\left(0, \sqrt{\sum_{l=l}^{j} l_{jl}^2}\right). \tag{A.5}$$

On the other hand, (A.6) is also valid, because it is a condition derived from the Cholesky decomposition, when it is applied to a correlation matrix, which has all the diagonal elements equal to 0

$$\sum_{l=l}^{j} l_{jl}^2 = 1. \tag{A.6}$$

Therefore, in the particular case $\mu_1 = \mu_2 = \ldots = \mu_l = 0$ y $\sigma_1 = \sigma_2 = \ldots = \sigma_l = 1$, y_{ji} will be distributed by a $N(0,1)$.

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