**Abstract**— Today’s power delivery infrastructure is operating under extreme stress. The last few decades have seen minimal investment in infrastructure, and little has been done to prepare the ageing equipment for increasing load growths and the demands of open access. Consequently, the deployment of Distributed Energy Resources (DER) is becoming an increasingly attractive alternative to the expensive and time consuming processes of upgrading and augmenting the transmission and distribution systems.

DERs are small, modular sources (generation or storage) of energy which are often more efficient and controllable than traditional power plants. These devices will be installed at or near centers of utilization. Eventually, as their penetration increases considerably, they will be interconnected in a grid-like fashion for stability and enhanced reliability. These grids are called ‘microgrids.’

This paper presents a rational method of building microgrids optimized for cost and subject to reliability constraints. The method is based on dynamic programming and consists of determining the optimal interconnection between microsources and load points, given their locations and the rights of way for possible interconnections. A new approach, called ‘unit link addition,’ is also introduced. The method is demonstrated using a 22-bus system.

**I. INTRODUCTION**

The significance of system planning and expansion has increased manifold during the past few years because of the radical changes going on in the power industry [1]. The system is being restructured. Deregulation is allowing open access on transmission lines and wheeling of large amounts of power from geographical distant areas. The transmission system which was originally designed for stability is now experiencing severe stress. Further, the aging infrastructure is no longer capable of accommodating load growth. Recent blackouts have pointed to the fact that the system is indeed experiencing unprecedented amounts of stress.

One possible solution is to build new transmission facilities. However, this is a very expensive and time consuming process. The other alternative is to use on-site generation. This is in the form of distributed generation, that can be used at or near points of utilization. This method is more attractive because firstly, it alleviates the need for building the costly transmission grid. Secondly, because of their very nature, distributed generators can be installed on demand, instead of waiting for a long time to build traditional plants. In this regard, there have been many efforts from both the government and industrial agencies for developing and deploying these resources. In particular, the US Department of Energy (USDoE) has laid out a strategic plan [2] [3] for effective placement and utilization of DERs in the form of microgrids.

As mentioned previously, DERs are likely to be sited at load points, i.e., in the distribution network. This integration will significantly change the structure of these networks. Traditionally distribution networks have been radial, and the focus of planning methodologies has been primarily on preserving the radiality of the distribution system [4] [5]. However, as the penetration of DERs increases, it may be prudent to interconnect these generating devices to form a grid. This would be an implementation of a microgrid.

Significant research has been conducted in the areas of transmission expansion planning [6] and developing microgrids [7] – [12]. Most notable is the “CERTS Microgrid Concept” which focuses on a self-sustained heat and power supply to a compact cluster of loads. Many of the approaches have focused on the optimal placement of DERs in the distribution network [11] [12]. These approaches have assumed and have attempted to preserve the fundamentally radial structure of the distribution grid. In contrast, we address the issue of distribution network expansion with a reliability criterion using deployment of distributed resources. Our approach is directed toward the development of microgrids that are networked in structure and conform to the US Department of Energy’s vision of microgrids that can operate in both grid-connected and islanded modes.

This paper presents a rational method of building cost optimal microgrids subject to reliability constraints. This work is an initial step towards fulfilling an immediate need for system planning tools that take into account the changes going on in the industry. It will enable design of reliable architectures for future system planning.

**II. SYSTEM MODELING**

1) Generators: These are modeled as two-state devices. Each generator $i$ is described by its maximum generating capacity $G_{max,i}$, and its forced outage rate $FOR_i$. 

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**Microgrid Architecture:**

**A Reliability Constrained Approach**

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2) Load: A single load scenario is considered; the load being at the peak value at each bus.

3) Transmission Lines: The basic element of the transmission network is the unit-link which is a transmission line with the following characteristics:
   - The unit-link has a fixed capacity called the unit-capacity, denoted by Cap_{u}.
   - Each unit link is of unit length and unit impedance. The unit-link corresponding to any two nodes i and j, denoted by unit-link-ij, has a length equal to the distance between the nodes. Hence we have a unit-cost of linking two nodes i and j given by Cost_{u_{ij}} = k \times l_{ij}, where k is a constant. Costs due to other factors may be added to the above to get the total cost of laying a link of unit capacity between those two nodes. Further the impedance of the unit-link between the same nodes is z \times l_{ij} where z is the unit-impedance.

4) Network Model: A linearized network model in the form of DC Load Flow has been used in this work.

III. PROBLEM FORMULATION

This work aims at determining the optimal network configuration which satisfies a minimum reliability requirement. Mathematically, this problem can be posed as:

Minimize:

\[ J = \sum_{i<j} J_{ij} \times x_{ij} \quad \text{for} \ 1 \leq i < j \leq N_n \]  

subject to:

\[ EIR > R_0 \]  

where,

\[ J \] = cost of the transmission network  
\[ J_{ij} \] = cost of interconnecting nodes i and j  
\[ x_{ij} \] = selection status of link i-j  
\[ N_n \] = number of nodes in the system  
\[ EIR \] = Energy Index of Reliability, and  
\[ R_0 \] = minimum required reliability.

This problem is amenable to a stage-wise decomposition. In each stage one line is added to the existing network. Therefore dynamic programming becomes a suitable tool for the solution method. This approach with the concept of the unit-link gives the optimal network along with the capacity of each link.

IV. SOLUTION STRATEGY

The problem is decomposed into many stages, where in each stage the network is incremented by another unit-link. Before we describe the strategy, first we need to define the different structures of the DP.

A. The Dynamic Programming Formulation

Each stage represents the total number of unit links that have been added to the system. There are many configurations that utilize the same total number of unit-links. Each configuration has an associated cost and offers a value of reliability. This measure of reliability is chosen as the “DP state”.

States for the next stage are built by adding another unit-link. Each possible unit-link between any pair of nodes is an alternative. These are the “DP decisions”.

It is quite possible that in a given stage, different configurations can have the same value of reliability, though with different costs. Therefore, it is very important during the building of stages to check for duplicate states. Whenever a new state for the next stage is generated by testing a decision, a check is performed to see if any other state with the same value of reliability exists. If such a state exists and has a higher cost, then it is replaced with the newly generated state. In other words, this ensures that the configuration represented by a state in a given stage is the most optimal way of achieving the reliability it offers by adding as many unit links as depicted by the stage.

B. Reliability Evaluation

An energy index of reliability is chosen. Given a set of generating units, the expected minimum curtailment evaluated over contingencies up to first order is used as the measure of system reliability.

The core of the reliability evaluation module is an LP formulation based on DC power flow. The aim is to minimize the total curtailment for a given network, which is given by:

\[ \text{Loss of Load} = \min \left( \sum_{i=1}^{N_b} C_i \right) \]  

subject to:

\[ \hat{B}_\theta + G + C = D \]  
\[ G \leq G_{\text{max}} \]  
\[ C \leq D \]  
\[ bA_\theta \leq F_{\text{max}} \]  
\[ -bA_\theta \leq F_{\text{max}} \]  
\[ G, C \geq 0 \]  
\[ \theta \text{ unrestricted} \]

where

\[ N_b = \text{number of buses} \]  
\[ N_t = \text{number of transmission lines} \]  
\[ C = N_b\text{-vector of bus load curtailments} \]  
\[ C_i = i\text{-th element of} \ C, \ i.e., \ unsatisfied \ demand \ at \ bus \ i \]  
\[ D = N_t\text{-vector of bus demands} \]  
\[ G_{\text{max}} = N_b\text{-vector of available generation at buses} \]  
\[ F_{\text{max}} = N_t\text{-vector of forward flow capacities of transmission lines} \]
**F**_{r}^{\text{max}} = \mathbf{N}_{l}\text{-vector of reverse flow capacities of transmission lines} \\
\mathbf{G} = \mathbf{N}_{b}\text{-vector of dispatched generation at buses} \\
\mathbf{\theta} = \mathbf{N}_{b}\text{-vector of bus voltage angles} \\
\mathbf{b} = \mathbf{N}_{l} \times \mathbf{N}_{l}\text{ primitive (diagonal) matrix of transmission line susceptances} \\
\mathbf{A} = \mathbf{N}_{l} \times \mathbf{N}_{b}\text{ element-node incidence matrix} \\
\mathbf{\hat{A}} = \mathbf{N}_{b} \times \mathbf{N}_{b}\text{ augmented node susceptance matrix} \\
\mathbf{A}^{T} \mathbf{b} \mathbf{A}

The above minimization procedure is performed for each contingency. In this work, generation contingencies up to first order have been considered.

Let \( LO{L}_{i} \) be the loss of load obtained for the \( i^{th} \) contingency, with a probability of \( \text{prob}_{i} \). Then the expected power not served is given by:

\[
EPNS = \sum_{1}^{N_{c}} LO{L}_{i} \times \text{prob}_{i}
\]

where,

- \( EPNS \) = Expected Power Not Served
- \( N_{c} \) = Number of contingencies

The reliability of the network is then given by:

\[
EIR = 1 - \frac{EPNS}{DT}
\]

where,

- \( EIR \) = Energy Index of Reliability
- \( DT \) = Total Power Demand

Normally, the \( EIR \) is computed using the equation:

\[
EIR = 1 - \frac{EUE}{ET}
\]

where,

- \( EUE \) = Expected Unserved Energy
- \( ET \) = Total Energy Demand

Equation (7) becomes equivalent to (6) when the demand is constant over the period of interest. In this work, only the peak load at the buses has been considered which is adequate for planning purposes. So the demand power remains constant during the period of interest.

**C. Strategy**

Though the problem can be solved in a stage-wise fashion, it is only the final stage that we are interested in. Therefore, given the present stage, there is no need to store stages other than the current one, which is used to build the next stage. So, along with storing the system reliability, the DP state contains additional information regarding the network configuration. This approach eliminates the need for backtracking. Further, this is memory efficient as we do not have to save the previous stages. The network information stored in the DP state is minimal and the memory overheads arising from this is insignificant in comparison with the requirements for storing all the stages.

This problem is solved in two phases. In the first phase, DP stages are built sequentially until one of the states in the stage being built just crosses the threshold reliability. It is possible that in the last stage of this phase there are other states which represent a system configuration offering a higher reliability than the threshold reliability. From all such states, the state that gives the lowest reliability is chosen and marked as the reference state. This state is defined as the reference state, with reliability \( R_{\text{ref}} \) and cost \( J_{\text{ref}} \). Then, we proceed with Phase II.

In the second phase, all the states from the last stage of Phase I satisfying the following criteria are chosen:

1) Cost \( \leq J_{\text{ref}} \)
2) Reliability \( \leq R_{\text{ref}} \)

These states are collated into one stage which forms the first stage of the second phase. From this stage subsequent stages are built. However, in this phase, the building of stages for any trajectory ends if the cost of that trajectory is no longer less than \( J_{\text{ref}} \). In other words, pursuing the trajectory will offer no advantage than that provided by the reference state. During this procedure, all states that meet the threshold reliability criterion are identified. Among all such states, the state that has the lowest cost is the optimal state.

The configuration represented by the optimal state is the optimal solution.

**D. Algorithm**

The flowcharts for Phase-I and Phase-II are shown in Fig. 1 and Fig. 2 respectively.

**V. APPLICATION**

The above algorithm was applied to a 22-bus system. This system was derived from the distribution network at Bus-2 of the RBTS [14]. From the layout given in the single line diagram of the above mentioned network, rights of way between nodes and costs thereof were established. For the sake of demonstration, the cost of interconnecting nodes \( i \) and \( j \) was assumed to be proportional to the distance as shown in the layout. The load was assumed to be at the peak for all the buses. It was assumed that the location and size of the generating units are known beforehand. Further, typical values of forced outage rates were taken.

In this system, Bus-22 is the Point of Common Coupling (PCC). However, a worst case situation was analyzed where there is no assistance from the grid and the load is at the system peak.

The generation and load data is produced below in Table I and Table II respectively.

The capacity of each unit-link was taken to be 0.2 MW, with impedance 0.006 p.u. per mile.

The program was executed with a setting of maximum \( EPNS \) of 4% of the total demand. The resulting network is shown in Fig. 3. The network configuration is tabulated in Table III.
**Fig. 1.** Flowchart for Phase I

**Fig. 2.** Flowchart for Phase II
TABLE I
GENERATION DATA FOR THE 22-BUS SYSTEM

<table>
<thead>
<tr>
<th>Bus</th>
<th>Gen (MW)</th>
<th>PER</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>5.0</td>
<td>0.06</td>
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<tr>
<td>5</td>
<td>10.0</td>
<td>0.10</td>
</tr>
<tr>
<td>10</td>
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</tr>
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<td>5.0</td>
<td>0.10</td>
</tr>
<tr>
<td>22</td>
<td>7.5</td>
<td>0.10</td>
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TABLE II
LOAD DATA FOR 22-BUS SYSTEM

<table>
<thead>
<tr>
<th>Bus</th>
<th>Load (MW)</th>
<th>Bus</th>
<th>Load (MW)</th>
<th>Bus</th>
<th>Load (MW)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>9</td>
<td>1.8721</td>
<td>16</td>
<td>0.7500</td>
</tr>
<tr>
<td>2</td>
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<td>10</td>
<td>0.8668</td>
<td>17</td>
<td>0.7291</td>
</tr>
<tr>
<td>3</td>
<td>0.8668</td>
<td>11</td>
<td>0.8668</td>
<td>18</td>
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</tr>
<tr>
<td>4</td>
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</tr>
<tr>
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<tr>
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<td>14</td>
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<tr>
<td>7</td>
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<td>0.7500</td>
<td>22</td>
<td>0.7500</td>
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<tr>
<td>8</td>
<td>1.6279</td>
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</tbody>
</table>

TABLE III
RESULTING NETWORK CONFIGURATION FOR 22-BUS SYSTEM

<table>
<thead>
<tr>
<th>LINE</th>
<th>Cap (MW)</th>
<th>LINE</th>
<th>Cap (MW)</th>
<th>LINE</th>
<th>Cap (MW)</th>
</tr>
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<tbody>
<tr>
<td>1-18</td>
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<td>0.8</td>
<td>21-22</td>
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</tr>
<tr>
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<td>0.8</td>
<td>10-16</td>
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<tr>
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<td>5-7</td>
<td>0.8</td>
<td>8-11</td>
<td>1.6</td>
</tr>
<tr>
<td>15-22</td>
<td>0.8</td>
<td>1-19</td>
<td>0.8</td>
<td>10-17</td>
<td>0.6</td>
</tr>
<tr>
<td>3-5</td>
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<td>0.8</td>
<td>17-22</td>
<td>0.4</td>
</tr>
<tr>
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<td>0.4</td>
<td>1-17</td>
<td>0.4</td>
</tr>
<tr>
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</tr>
<tr>
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<td>0.2</td>
<td>16-22</td>
<td>0.2</td>
</tr>
<tr>
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<tr>
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<td>5-8</td>
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<tr>
<td>5-11</td>
<td>0.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

VI. CONCLUSIONS AND DISCUSSIONS

This paper presented a rational approach to designing microgrid architectures that optimized for cost and subject to reliability constraints.

A dynamic programming method was developed to determine the optimal interconnection between microsources and load points, given their locations and the rights of way for possible interconnections. To render the size of the problem manageable, a scheme was implemented that minimized the storage requirements. A new approach, called ‘unit link addition,’ was also introduced. The method was demonstrated using a 22-bus system.

The illustration provided in this work consisted of a case where a system was built from scratch. This case was selected due to its suitability for demonstration of the method, but realistically, a microgrid will rarely be built from scratch; it is more likely to evolve from an existing distribution system that is radial in nature. The method can as well be applied to this case, and would indeed be a more rational approach to enabling this evolution that ad hoc expansion in response to load growth.

The work reported here represents the authors’ first step in the direction of developing a methodology for rational design of microgrids. Considerable improvements over the work described are now under development and implementation. These include consideration of higher order contingencies, modeling of reliability differentiated service and refinement of the method to efficiently deal with the increased complexity. These developments will be reported in due course.

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REFERENCES


